15. THE EFFECT OF WAVEFRONT INTERACTIONS ON PATTERN FORMATION IN EXCITABLE MEDIA

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The nonlinear interactions of solitary wavefronts in excitable media are determined by the manner of recovery to the rest state. The distance between a pair of wavefronts tends to lock at one of countably many possible values in the case of oscillatory recovery, while it increases indefinitely when the recovery is monotonic. We derive these results from the basic reaction diffusion equations and study the implications on pattern formation in one and two space dimensions. In particular we demonstrate how spatiotemporal complexity may arise in one dimension, and discuss possible consequences of the interplay between wavefront-interactions and curvature in two dimensions.

1. Introduction

Excitable media provide good examples of nonequilibrium systems where spatially extended patterns can be understood in terms of simple building blocks: solitary waves or impulses in one space dimension, solitary wavefronts and vortices in two-dimensions and so on. The basic reason for that lies in the localized nature of these structures. Thus a given impulse in a wavetrain can be viewed as an independent, particle-like entity that propagates in the perturbative field of nearby impulses. Following further the analogy to particles we would like to know how to determine the degrees of freedom of localized structures, and how to obtain their time evolution in the presence of other structures in the neighbourhood. Once the dynamical laws of individual localized structures are known we can address the many body problem and study spatiotemporal behaviours of extended patterns.

The key to these questions rests in the continuous symmetries of the system. Assuming a homogeneous medium (or translational invariance), the degree of freedom of an impulse is the position of that impulse at a given time, \(x(t)\), or the time it passes through a given location, \(t(x)\). In the presence of a perturbing field this degree of freedom acquires a slow component, \(\chi(t) = x(t) - c_0 t\), where \(c_0\) is the propagation speed of the unperturbed impulse. An equation of motion of this component is derivable as a solvability condition which guarantees that the unperturbed form of the
impulse that propagates at speed \( c_0 + \chi \), is an approximate solution of the perturbed problem.

In section 2 I will show how this approach can be used to study the interaction between a pair of impulses and to understand the emergence of complex spatial structures in one space dimension. In section 3 I will proceed to patterns in two dimensions and consider dynamical aspects of rotating spiral waves. More specifically, I will show how the interplay between wavefront interactions and curvature may lead to destabilization of steady rotation and to spiral waves whose cores expand in time. A brief discussion of these results in section 4 will conclude this presentation.

2. Wavetrains of impulses in one space dimension

Consider homogeneous excitable media that support solitary waves propagating at constant speed \( c_0 \). These media are described by reaction-diffusion equations (rde's) of the general form [1]

\[
\frac{\partial}{\partial t} U = M(U) + D \frac{\partial^2}{\partial x^2} U,
\]

where \( U \) represents a set of fields, \( M(U) \) is the reaction part and \( D \) is a matrix of transport coefficients (diffusion, conduction, etc.). It is assumed that \( M(0) = 0 \). The solution \( U = 0 \) represents the quiescent state of the medium.

Let \( U = H(x - c_0 t) \) denote a solitary wave solution of (1). An alternative notation, \( U = S(t - x/c_0) \), will be useful when viewing time as the dependent variable. Being localized, the solitary wave solution decays off exponentially as \( x = x - c_0 t \to \pm \infty \) or as \( t = t - x/c_0 \to \infty \). The tail of the solitary wave \( (x \to -\infty \text{ or } t \to \infty) \) represents the manner in which the medium recovers to the rest state after excitation. Two principal forms of recovery are possible: monotonic, in which case the tail assumes a pure exponential form, \( H(x) \propto \exp(\eta L x) \), and (damped) oscillatory, \( H(x) \propto \exp(\eta L x)\cos(\nu L x + \phi L) \). In the latter case the system undergoes a succession of super and subnormal periods until complete recovery is attained [2,3]. The head of the solitary wave is assumed to have in both cases a pure exponential form, \( H(x) \propto \exp(-\eta R x) \) (\( x \to \infty \)).

Imagine now two impulses propagating in the positive \( x \) direction. If the spacing, \( \lambda \), between the two is considerably larger than their width, \( \eta L^{-1} \), each impulse can be viewed as propagating in the perturbative field of the other. In such a case we may try a solution in the form of a superposition of displaced solitary waves:

\[
U(x,t) = H(x - \chi_1(t)) + H(x - \chi_2(t)) + R,
\]

where \( R \) is a small correction term. Equations of motion for the displacements, \( \chi_1, \chi_2 \), follow from solvability conditions which remove singularities from \( R \). Expressed in terms of the impulse positions, \( x_i = c_0 t + \chi_i \), these equations read [4]