In this chapter we will cover three general applications of Fisher information in the analysis of financial economics. The first two applications (Sections 2.1 and 2.2) demonstrate how constraints based on knowledge of system data can be used to construct probability laws. This is by the use of a form of extreme physical information (EPI) known as minimum Fisher information (MFI). The third application (Section 2.3) shows how optimum investment strategies can arise out of the application of EPI to a financial system. That is, a dynamical investment program that enforces an optimization of information flow, achieving $I - J = \text{extremum}$, can also, in certain cases, achieve a program of optimal capital investment.

2.1. Constructing Probability Density Functions on Price Fluctuation

2.1.1. Summary

Information flow is both central to economic activity [1–3] and a primary causal factor in the emergence, stability, and efficiency of capital markets [4–8]. The well-known interaction between information and economic agents in the price discovery process suggests that information flow may also play a central role in determining the dynamical laws of an economic system. Our view is that this determination arises out of perturbation of both the perceived and intrinsic information levels of the economic system. Specifically, an economic agent perturbs both (i) the Fisher information $I$ about system value that is based solely on observations of the system and (ii) the Fisher information $J$ about system value that is fundamental to the system. Thus, $I - J$ represents the difference between the perceived and intrinsic values of the system, and it is this difference that (a) implies an opportunity for investment and (b) is perturbed by the economic agent. If the perturbations $\delta I$ and $\delta J$ are equal, then $\delta(I - J) = 0$, $I - J$ is an extremum, and we can employ the variational approach known as extreme physical information [9] to determine the dynamical laws of the economic system. In this way the dynamical laws arise out of an analysis of the flow of information.
about investment value and the opportunities for investment that these flows engender.

We begin this section with the construction of *equilibrium* distributions, and illustrate this in the construction of yield curves. The framework that we develop for understanding the structure of equilibrium probability laws will form the basis for the second general application: the construction of *dynamics* (Section 2.2). In this section we will see that the equilibrium distributions are but the lowest order modes in a multimode system. The higher order modes give rise to relaxation dynamics: A financial realization of this structure is the model structure of yield curves as below.

### 2.1.2. Background

In this section, the focus of the research is upon the structure of financial economics, as defined by its probability density functions (PDFs) $p(x)$ on price valuation $x$. Historically, much knowledge of economic structure and dynamics has arisen by taking a physical perspective. The resulting “econophysics” has emerged as a distinct field of physics. Its basic premise is that the same phenomenological and mathematical insights that are used to provide unification for complex *physical* systems can also apply to problems of economics and finance. A now-classic example of this cross-fertilization is *Brownian motion*, first analyzed by Louis Bachelier in his Ph.D. thesis [10]. Rediscovered in the mid-twentieth century by workers such as Osborne [11], econophysics research achieved a high point with the discovery of the celebrated option pricing equation by Black, Scholes, and Merton [13,14], which created the profession of financial engineering. The current state of this field is well represented by a number of substantial texts [15–17]. These, together with the uniquely informed perspective on econometrics [18], form what might be considered the current basis of this field.

### 2.1.3. Variational Approaches to the Determination of Price Valuation Fluctuation

Variational approaches have been found useful for determining price fluctuation curves $p(x)$. As with much of the work applying statistical mechanics to finance, initial forays in this direction were based on the application of Shannon entropy [19], as popularized by Jaynes [20], to problems in financial economics [21,22]. Also, a principle of maximum entropy has found application as a useful and practical computational approach to financial economics [22–26].

However, an alternative approach arises out of the *participatory*, or human, component of any financial or economic system. Recall Wheeler’s principle of a “participatory universe” ([27,28], Section 1.4.4):

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1 A remarkable collection of papers from that time, including Osborne’s, can be found in Cootner [12].