Algorithmic graph problems constitute approximately one third of the problems in this catalog. Problems from other sections could have been formulated equally well in terms of graphs, such as bandwidth minimization and finite-state automata optimization. Identifying the name of a graph-theoretic invariant or problem is one of the primary skills of a good algorist. Indeed, the catalog will tell you exactly how to proceed as soon as you figure out your particular problem’s name.

In this section, we deal only with problems for which there are efficient algorithms to solve them. As there is often more than one way to model a given application, it makes sense to look here before proceeding on to the harder formulations.

The algorithms presented here have running times that grow polynomially with the size of the graph. We adopt throughout the convention that \( n \) refers to the number of vertices in a graph, while \( m \) is the number of edges.

Graphs are often best understood as drawings. Many interesting graph properties follow from the nature of a particular type of drawing, such as planar graphs. Thus, we also discuss algorithms for drawing graphs, trees, and planar graphs.

Most advanced graph algorithms are difficult to program. However, good implementations are available if you know where to look. The best general sources include LEDA [MN99] and the Boost Graph Library [SLL02]. However, better special-purpose codes exist for many problems.

See the *Handbook of Graph Algorithms* [TNX08] for up-to-date surveys on all areas of graph algorithms. Other excellent surveys include van Leeuwen [vL90a], and several chapters in Atallah [Ata98]. Books specializing in graph algorithms include:

- *Sedgewick* [Sed98] – The graph algorithms volume of this algorithms text provides a comprehensive but gentle introduction to the field.
• Ahuja, Magnanti, and Orlin [AMO93] – While purporting to be a book on network flows, it covers the gamut of graph algorithms with an emphasis on operations research. Strongly recommended.

• Gibbons [Gib85] – A good book on graph algorithms, including planarity testing, matching, and Eulerian/Hamiltonian cycles, as well as more elementary topics.

• Even [Eve79a] – An older but still respected advanced text on graph algorithms, with a particularly thorough treatment of planarity-testing algorithms.