Chapter 10

Games and Strategies

You have to learn the rules of the game. And then you have to play better than anyone else.

- Albert Einstein.

Games-of-chance derive this title from the fact that luck plays a part in deciding the winner of a play of the game. Sometimes the game consists solely of luck, as with Coin-Flipping ("heads wins") or Card-Cutting ("highest card wins"). Typically, though, this isn’t the case, and a sensible strategy is needed to beat a good player who isn’t burdened by a string of extraordinarily bad deals of the cards (in the case of, e.g., Poker or Bridge), or throws of the dice (in the case of, e.g., Backgammon or Monopoly). However, casinos operate (very successfully!) on the premise that (most of) their clientele do not play with luck on their side.

What we might call games-of-no-chance are those games for which the winner is decided based solely on ability. Examples of such games are Chess and Go. They involve no decisions taken on the results of random events such as the deal of cards or the throw of dice, and no information is hidden from the players (apart from what moves the other player will choose to make during the play of the game).

For example, in the children’s paper-and-pencil game Noughts and Crosses (also known as Tic-Tac-Toe), two players alternately place crosses (×) and noughts (○) in nine square spaces arranged in a 3 × 3 grid. The goal of the first player is to align three crosses in a line (row, column or diagonal), and the goal of the second player is to align three noughts in a line (row, column or diagonal). A player wins the game if they achieve their goal before the other player does so. A game that ends with a full grid without a line of crosses or noughts is a draw.

When children first learn to play this game, the outcomes will be variable; sometimes the first player wins, sometimes the second player wins, and sometimes the game ends in a draw. However, every child eventually becomes bored with this game, as they discover that they can only win if their opponent makes a silly error. This is regardless of whether they are
playing as first player or second player, though it seems that the first player
should have a distinct advantage.

10.1 Strategies for Games-of-No-Chance

In this chapter we shall be interested in such two player games-of-no-chance. We shall typically refer to the first player as A (for Alice) for whom we shall use female pronouns (she, her), and the second player as B (for Bob) for whom we shall use male pronouns (he, his). Furthermore, these will be games of perfect information, meaning that both players will be aware of all aspects of the game: at every point in the game, both players know what moves have been made up to that point in time, as well as what moves their opponent can make in response to any move that they themselves make. The game of PAPER-SCISSORS-ROCK, for example, is not a game of perfect information, as neither player has information regarding the move being made by the other player. While there is no element of chance in the players’ decision making, as each player is free to choose whatever move they wish, the lack of information about the opponent’s move makes luck a factor in this game.

Another typical feature of the games that we shall consider is finiteness. A finite game is one that is guaranteed to terminate within a finite number of steps. This isn’t true of many games, for example CHESS (unless some rule is introduced which declares a game to be a draw if it continues indefinitely, the standard rule being that a draw is declared if 50 consecutive moves have been made without a piece being captured nor a pawn being moved). If a play of a particular game may continue indefinitely, we will rule infinite plays to be predetermined in some way; that is, either the first player wins every infinite play, or the second player wins every infinite play, or the game is declared to be a draw. For example, we may declare that every infinite play of the game of CHESS is ruled to be a draw.

We shall at times consider games in which the first player is in the role of an attacker; she makes attacking moves which the second player, in the role of a defender, must defend against with his responses. We may refer to such games as attacker-defender games. The first player’s aim is to achieve some goal (which will end the game), while the second player’s aim is to prevent her from doing this. The important aspect of these games is that a play which continues forever is a positive result for the second player; that is, every infinite play of an attacker-defender game is ruled to be a win for the second player.

A strategy for a player in a game is a rule which tells that player what move to make each time it is their turn to move. A strategy which guarantees that you will win the game regardless of what moves your opponent