Chapter 2
Foundations of Mechanism Design

This chapter forms the first part of the monograph and presents key concepts and results in mechanism design. The second part of the monograph explores application of mechanism design to contemporary problems in network economics. The chapter comprises 21 sections that can be logically partitioned into four groups. Sections 2.1 through 2.5 constitute Group 1, and they set the stage by describing essential aspects of game theory for understanding mechanism design. The five sections deal with strategic form games, dominant strategy equilibria, pure strategy Nash equilibria, mixed strategy Nash equilibria, and Bayesian games. Sections 2.6 through 2.12 constitute the next group of sections, and they deal with fundamental notions and results of mechanism design. The sections include a description of the mechanism design environment, social choice functions, implementation of social choice functions by mechanisms, incentive compatibility and revelation theorem, properties of social choice functions, the Gibbard–Satterthwaite impossibility theorem, and the Arrow’s impossibility theorem. Following this, the sections in the third group (Sections 2.13 - 2.20) present useful mechanisms that provide the building blocks for solving mechanism design problems. The sections here include: The quasilinear environment, Groves mechanisms, Clarke mechanisms, examples of VCG mechanisms, the dAGVA mechanism, Bayesian mechanisms in linear environment, revenue equivalence theorem, and optimal auctions. Finally, in Section 2.21, we provide a sprinkling of further key topics in mechanism design. The chapter uses a fairly large number of stylized examples of network economics situations to illustrate the notions and the results.

2.1 Strategic Form Games

Game theory may be defined as the study of mathematical models of interaction between rational, intelligent decision makers [1]. The interaction may include both conflict and cooperation. The theory provides general mathematical techniques for analyzing situations in which two or more individuals (called players or agents) make decisions that influence one another’s welfare. There are many categories of games that have been proposed and discussed in game theory. We introduce here a class of games called strategic form games or normal form games, which are most appropriate for the discussions in this monograph. We start with the definition of a strategic form game.
Definition 2.1 (Strategic Form Game). A strategic form game $\Gamma$ is defined as a tuple $\langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$, where $N = \{1, 2, \ldots, n\}$ is a finite set of players; $S_1, S_2, \ldots, S_n$ are the strategy sets of the players $1, \ldots, n$, respectively; and $u_i : S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R}$ for $i = 1, 2, \ldots, n$ are mappings called the utility functions or payoff functions.

The strategies are also called actions or pure strategies. We denote by $S$, the Cartesian product $S_1 \times S_2 \times \cdots \times S_n$. The set $S$ is the collection of all strategy profiles of the players. Note that the utility of an agent depends not only on its own strategy but also on the strategies of the rest of the agents. Every profile of strategies induces an outcome in the game. A strategic form game is said to be finite if $N$ and all the strategy sets $S_1, \ldots, S_n$ are finite.

The idea behind a strategic form game is to capture each agent’s decision problem of choosing a strategy that will counter the strategies adopted by the other agents. Each player is faced with this problem and therefore the players can be thought of as simultaneously choosing their strategies from the respective sets $S_1, S_2, \ldots, S_n$. We can view the play of a strategic game as follows: each player simultaneously writes down a chosen strategy on a piece of paper and hands it over to a referee who then computes the outcome and the utilities. Several examples of strategic form games will be presented in Section 2.1.2.

2.1.1 Key Notions

There are certain key notions underlying game theory. We discuss these notions and a few related issues.

2.1.1.1 Utilities

Utility theory enables the preferences of the players to be expressed in terms of payoffs in some utility scale. Utility theory is the science of assigning numbers to outcomes in a way that reflects the preferences of the players. The theory is an important contribution of von Neumann and Morgenstern, who stated and proved in [2] a crucial theorem called the expected utility maximization theorem. This theorem establishes for any rational decision maker that there must exist a way of assigning utility numbers to different outcomes in a way that the decision maker would always choose the option that maximizes his expected utility. This theorem holds under quite weak assumptions about how a rational decision maker should behave.