Chapter 14
Financial Applications: Parallel Portfolio Optimization

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Abstract
Portfolio optimization is an area of tremendous importance for long-term investors. It is concerned with the problem of how to best diversify investment into different classes of assets (such as stock, bonds, real estate, and options) in order to meet liabilities and to maximize the expected surplus, while hedging against unacceptable risk.

Different formulations of the problem have been suggested over time, starting from Harry Markowitz’ seminal mean-variance model. Practical and theoretical limitations of the mean-variance model have led to the proposal of different utility functions, risk measures, and dynamic multi-period models that allow rebalancing of the portfolio to hedge against adverse market conditions. Furthermore new legislation has often resulted in the necessity to introduce new classes of constraints on the portfolio composition.

Multi-period portfolio optimization problems are usually treated as stochastic programming problems, that is, they involve optimization over a selection of future scenarios. The desirability of having future scenarios match static and dynamic correlations between assets for all future time periods leads to problems of truly enormous sizes (often reaching millions of unknowns or more). Clearly parallel processing becomes mandatory to deal with such problems.

The most popular solution techniques for stochastic programming problems are decomposition methods and interior point methods (IPMs). Both approaches lend themselves to parallel implementations and impressive results have been achieved here in the past few years. We will review different implementations on a variety of computing platforms ranging from dedicated parallel machines to PC clusters all the way to grid environments. Typically the solution and parallelization techniques have
to be adapted to both the specific model formulation and the available computing platform.

14.1 Introduction

Portfolio selection is one of the most relevant and most studied topics in finance. The problem, in its basic formulation, is concerned with balancing the twin contradictory objectives of maximizing return of investment while minimizing the associated risk. Early models arising from Markowitz’ seminal work [1] are static and deterministic. They consider a set $\mathcal{A}$ of possible investments and assume that joint distribution of asset returns is multivariate normal $N(\mu, \Sigma)$ with known means $\mu$ and covariance matrix $\Sigma$. The objective is to maximize expected single period return, while bounding the variance of the portfolio return as a measure of risk exposure.

There are a number of perceived weaknesses with this approach that have emerged over the years: the assumption of normal asset returns (neglecting observed “fat tails”), the assumption of known fixed means and covariances (which have to be estimated from historical data, and are clearly neither known exactly nor constant over time), and not least the inability to capture dynamic effects such as transaction costs and the possibility to hedge risk through rebalancing of the portfolio at future time stages. In the past 20 years emphasis has shifted toward stochastic dynamic models that allow the adequate representation of non-normal joint return distribution and the effects of portfolio rebalancing. An overview of these issues is given in the review paper [2].

Realistic models need to account for long planning horizons and adequate capturing of the joint distributions of all future events that can influence the return of the portfolio over the whole planning horizon. These requirements quickly result in astronomical problem sizes. While general advances in the power of desktop computers have made larger problem formulations tractable, the area is a prime candidate for the successful use of parallel algorithms. This is expected to be the case even more in future as the trend for desktop and laptop computers is increasingly to multicore architectures.

The major applications for dynamic portfolio optimization are Asset and Liability Management (ALM) models in which the investor seeks an optimal long-term investment policy that meets anticipated (but unknown) liabilities and maximizes the expected surplus return, while minimizing the risk of defaulting on the liability payments. This is a model of prime importance to long-term investors such as insurances and pension funds.

In the following section we will review various formulations of the ALM model that have been proposed in the literature and discuss their properties, in particular in view of parallel solution approaches. Sections 14.3, 14.4, 14.5, and 14.6 describe popular parallelizable solution approaches to the models such as decomposition methods (Sect. 14.3), (Sect. 14.4), IPMs and evolutionary algorithms (Sect. 14.5).