Chapter 5
Robust Disturbance Attenuation for Uncertain Networked Control Systems

5.1 Introduction

For time-delay problems encountered in engineering, two approaches are employed. One is to obtain information on the time-delay and subsequently to use this information to solve the problem. The other is to attenuate the effects caused by delay disturbances when the delays cannot be effectively used or obtained. Furthermore, the problem of performance control with disturbance attenuation for time-delay systems has gathered much attention in recent years [73, 80, 81].

The $\mathcal{H}_\infty$ control problem is able to address the issue of system parameter uncertainty, and also be applied to the typical problem of disturbance input control. It was initially formulated [82] in the early 1980s [83] where the $\mathcal{H}_\infty$ norm plays an important role and resulted from the requirement of disturbance attenuation characterized by the $L_2$ gain. The effectiveness of the controller in attenuating according to the $\mathcal{H}_\infty$ norm has been widely reported and intensively studied for systems without input delays.

Recently, there have been interesting studies investigating the design of the $\mathcal{H}_\infty$ controller to guarantee not only asymptotic stability but also the $\mathcal{H}_\infty$ norm bound of a closed-loop system with time-delays. Based on the solution of a Riccati-like equation, a method to obtain the gain matrix of the $\mathcal{H}_\infty$ controller of linear systems with constant delays was proposed in [84]. In [85], the authors consider the $\mathcal{H}_\infty$ controller design problem for linear systems with time-varying delays in states. In [87] the robust $\mathcal{H}_\infty$ performance for linear delay differential systems is studied with an uncertain constant time-delay and time-varying norm-bounded parameter uncertainties.

The aforementioned results are mostly obtained for systems with state delays. Due to the characteristics of communication networks, network-induced time-delays are input delays. So far, performance control with disturbance attenuation has not been addressed for systems with uncertain time-varying pure input delays.

We attempt to solve this problem in this chapter. It should be noted that for systems with time-varying input delays, it is difficult to analyze disturbance attenuation based on the gain characterization, because the state variation depends not only on
the current but also the history of exterior disturbance input. In this chapter, a generalized disturbance attenuation will be introduced. This generalized disturbance attenuation reduces to the standard disturbance attenuation characterized by the $L_2$ gain when the delay is zero. In the light of such formulation, our object is to design a dynamic output feedback controller such that both robust stability and a prescribed disturbance attenuation performance for the closed-loop NCS are achieved, irrespective of the uncertainties and network-induced effects, i.e., network-induced delays and packet dropouts in both the sensor-controller and controller-actuator channels. Based on the Lyapunov–Razumikhin method, the existence of a delay-dependent controller is given in terms of the solvability of BMIs.

5.2 System Description and Problem Formulation

Assume that the uncertain linear continuous state-space model of the plant dynamics is described by the following equations:

\[
\begin{align*}
\dot{x}(t) &= (A + \Delta A)x(t) + (B_1 + \Delta B_1)w(t) + (B_2 + \Delta B_2)u(t) \\
z(t) &= (C_1 + \Delta C_1)x(t) + (D_1 + \Delta D_1)u(t) \\
y(t) &= (C_2 + \Delta C_2)x(t) + (D_2 + \Delta D_2)w(t)
\end{align*}
\]  

(5.1)

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^p$ is the exogenous disturbance input and/or measurement noise, $y(t) \in \mathbb{R}^l$ and $z(t) \in \mathbb{R}^s$ denote the measurement and regulated output respectively. Matrices $A$, $B_1$, $B_2$, $C_1$, $C_2$, $D_1$, and $D_2$ are of appropriate dimensions.

Matrices $\Delta A$, $\Delta B_1$, $\Delta B_2$, $\Delta C_1$, $\Delta C_2$, $\Delta D_1$, and $\Delta D_2$ characterize the uncertainties in the system and satisfy the following assumption:

**Assumption 5.1.**

\[
\begin{align*}
\begin{bmatrix} \Delta A & \Delta B_1 & \Delta B_2 \end{bmatrix} &= H_1 F(t) \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix}, \\
\begin{bmatrix} \Delta C_1 & \Delta D_1 \end{bmatrix} &= H_2 F(t) \begin{bmatrix} E_1 & E_3 \end{bmatrix}, \\
\begin{bmatrix} \Delta C_2 & \Delta D_2 \end{bmatrix} &= H_3 F(t) \begin{bmatrix} E_1 & E_2 \end{bmatrix},
\end{align*}
\]

where $H_1$, $H_2$, $H_3$, $E_1$, $E_2$, and $E_3$ are known real constant matrices of appropriate dimensions, and $F(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F(t)^TF(t) \leq I$, in which $I$ is the identity matrix of appropriate dimension.

The overall system setup to be investigated is depicted in Figure 5.1. Following the same lines in Chapter 2 with regard to the modeling of NCSs, a dynamic output feedback controller is constructed at time $t$ as follows:

\[
\text{Controller } \mathcal{G} : \begin{align*}
\dot{x}(t) &= \hat{A}(\eta_1(t), \eta_2(t))\hat{x}(t) + \hat{B}(\eta_1(t), \eta_2(t))y(t - \tau(\eta_1(t), t)), \\
u(t) &= \hat{C}(\eta_1(t), \eta_2(t))\hat{x}(t),
\end{align*}
\]  

(5.2)