Chapter 11
Fractional-order Mathematical Model of Pneumatic Muscle Drive for Robotic Applications

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11.1 Introduction

Pneumatic muscles are becoming increasingly attractive robotic drives for those applications in which stiffness of a kinematic chain have to be regulated. The stiffness coefficient of the muscle depends mainly on the level of pressure and its initial tension. These values describe the point of work on the static characteristics of the drive. Such a drive could be treated as a regulated spring. However, from the dynamic point of view the muscle is something more than a simple spring. Generally, three-element models (of the type R, L, C) are used in the literature to present the dynamical effects that characterise the drive [6]. It leads to second order differential equations that describe the features of the single muscle or the whole drive in the neighborhood of the static point of work. Sometimes this simple model is not sufficient and some phenomena of the muscle are modeled by additional inertial elements of the first or second order, and the model is characterized by three or four parameters. Precisely speaking, the muscle has to be treated as an element with distributed parameters and to describe its phenomena, partial differential equations could be used. Unfortunately, such an analytical model does not exist in a general case. A finite element method can be applied to derive a numerical model convenient for specific purposes [10].

The methodology proposed in the present paper allows finding a model of medium complexity that could be placed in between the two treatments mentioned above. It combines the simplicity of a lumped parameter model with some of the complicated processes hidden in the muscle, like nonlinearity of friction, braided shell extension, adiabatic processes and so on. The fractional calculus as a complementary mathematical tool to commonly known differential calculus is becoming very popular in different research areas, ranging from electrical engineering,
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electromagnetism, electrochemistry, electronics, mechanics, rheology to biophysics and economy [1, 2, 3, 4, 5, 8]. It seems to be very useful in complex unmodelable processes, for instance, in describing the transient behavior of a pneumatic muscle drives.

11.2 Pneumatic Muscle Description

The static characteristic of a single muscle is a function of a pressure \( p \) inside the muscle and its contraction \( \varepsilon \), according to formula [9]

\[
f(p, \varepsilon) = \frac{\pi r_0^2}{\sin^2(\alpha_0)} p \left(3(1 - \gamma \varepsilon)^2 \cos^2(\alpha_0) - 1\right)
\]  

(11.1)

where \( r_0 \) is a nominal radius of a cross-section, \( \alpha_0 \) is the angle of net in a braided shell, and \( \gamma = 1.25 \div 1.35 \) is a coefficient that correspond to deformation of both ends of the muscle from its theoretical cylindrical shape. Contraction is defined as the shortening of the muscle \( l_0 - l \) related to its nominal length, \( \varepsilon = (l_0 - l)/l_0 \). The drive system of a single robotic joint is usually based on antagonistic work of a pair of two symmetric muscles, as shown in Fig. 11.1. From formula (11.1) it follows

\[
\tau = \kappa_1(p_1 - p_2) + \kappa_2 \theta^2(p_1 - p_2) - \kappa_3(p_1 + p_2)\theta
\]  

(11.2)

where \( \kappa_1, \kappa_2 \) and \( \kappa_3 \) depend on parameters of the muscles, according to formulae

\[
\kappa_1 = \frac{\pi r_0^2}{\sin^2(\alpha_0)} p \left(3(1 - \gamma \varepsilon_0)^2 \cos^2(\alpha_0) - 1\right),
\]

\[
\kappa_2 = \frac{3\pi r_0^2 R^3 \gamma^2}{\tan^2(\alpha_0)}, \quad \kappa_3 = \frac{6\pi r_0^2 R^2 \gamma}{\tan^2(\alpha_0)}(1 - \gamma \varepsilon_0)
\]

(11.3)

where \( \varepsilon_0 \) is the contraction of the muscles at a static point of work.