Chapter 4

Life’s Still Lifes

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The de Bruijn diagram describing those decompositions of the neighborhoods of a one dimensional cellular automaton which conform to predetermined requirements of periodicity and translational symmetry shows how to construct extended configurations satisfying the same requirements. Similar diagrams, formed by stages, describe higher dimensional automata, although they become more laborious to compute with increasing neighborhood size. The procedure is illustrated by computing some still lifes for Conway’s game of Life, a widely known two dimensional cellular automaton.1

4.1 Introduction

Public attention was drawn to cellular automata by Martin Gardner’s monthly column Mathematical Recreations, a regular feature of Scientific American for many years. The October, 1970, issue [2] featured the game of Life, which had been invented about that time by the British mathematician John Horton Conway. Sufficient interest was aroused by the game for it to be followed up in several later columns, and to support a newsletter [8] for nearly three years. Gardner’s columns have now been collected into one of the compilations that are regularly published by W.H. Freeman and Company [3], while Conway’s own version of the game is available in the recent Academic Press book [1] Winning Ways.

However, there had been much previous interest in cellular automata, beginning at least with the work [5] of Warren McCulloch and Walter Pitts on neural nets, later including John von Neumann’s investigations [7] into self reproduction and automatic factories. Interest still continues, a recent example being Stephen Wolfram’s examination [10] of one dimensional automata from the point of view of chaos in complex systems theory.

1This paper is written in September 10, 1988.
One of the fundamental expectations in the theory of automata is that the automaton will eventually settle down into a fixed cycle of states, which will then characterize its long term behavior. Some modification of this principle must be expected for infinite automata, nevertheless the search for states of low period constituted an important part of the activity inspired by the announcement of *Life*. Someone with a crystallographer’s frame of mind might well have undertaken a classification of all such states, beginning with those of period 1, which Conway called “still lifes.” This article describes how such a classification can be obtained.

### 4.2 Cellular Automata

Mathematically, an automaton consists of a set of states, together with a set of mappings of the state set into itself. Each mapping is identified with a signal, which is supposed to cause a change of state. Signals can therefore be considered as inputs to the automaton, which in turn could be considered as a neural net, an electronic circuit, or some other structure. In that case, outputs might also considered, and altogether the groundwork has been laid for some kind of fundamental theory of computation, or at least of computing devices. Much of the theory of automata proceeds in that direction.

Cellular automata are those for which a large number of similar automata — the cells — are connected together in some regular pattern, and for which the signals are the information which each cell has concerning some of its neighbors, most likely including self-awareness. From time to time the cells change their state, according to this knowledge. McCulloch and Pitts would have the connectivity of the cells modelling some physiological system, but lacking definite structures to follow, the tendency has been to use crystallographic lattices of low dimension. Von Neumann worked with two dimensions, which was also the arena for Conway’s game.

*Life* presupposed binary cells occupying a two dimensional square lattice, the neighborhood of each cell consisting of itself, four lateral neighbors, and four diagonal neighbors; a total of nine cells altogether. Many other combinations are possible, but Conway chose one of them, as well as a particular rule of transition, for his game after discarding many alternatives. Adopting his picturesque ecological metaphor, binary cells are either dead or alive; in each generation,

- new cells are born to three live “parents”
- old cells survive if they have two or three live neighbors
- all other cells either die or remain dead

There are $2^9$, or 512, different combinations of dead and live neighbors. Each combination can evolve in its own way, giving the enormous number of $2^{29}$ different rules, or games, which Conway could have chosen; nevertheless that one choice has lived up to his expectations of finding an interesting game. Part of the choice consisted in selecting a symmetric rule; it is reasonable to suppose that a square