Chapter 7
The B36/S125 “2x2” Life-Like Cellular Automaton

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The B36/S125 (or “2x2”) cellular automaton is one that takes place on a 2D square lattice much like Conway’s Game of Life. Although it exhibits high-level behaviour that is similar to Life, such as chaotic but eventually stable evolution and the existence of a natural diagonal glider, the individual objects that the rule contains generally look very different from their Life counterparts. In this article, a history of notable discoveries in the 2x2 rule is provided, and the fundamental patterns of the automaton are described. Some theoretical results are derived along the way, including a proof that the speed limits for diagonal and orthogonal spaceships in this rule are \( c/3 \) and \( c/2 \), respectively. A Margolus block cellular automaton that 2x2 emulates is investigated, and in particular a family of oscillators made up entirely of \( 2 \times 2 \) blocks are analyzed and used to show that there exist oscillators with period \( 2^\ell (2^k - 1) \) for any integers \( k, \ell \geq 1 \).

7.1 Introduction

One cellular automaton that has drawn a fair amount of interest recently is the one that takes place on a grid like Conway’s Game of Life, except that dead cells are born if they have 3 or 6 live neighbours, and alive cells survive if they have 1, 2, or 5 live neighbours. Its birth and survival information is conveyed by the rulestring “B36/S125”. This rule exhibits many qualities that are similar to those of Life — for example, evolution seems unpredictable and random patterns tend to evolve into “ash fields” consisting of several small stable patterns (known as still lifes) and periodic patterns (known as oscillators).

The B36/S125 automaton has become known as “2x2” because of the fact that it emulates a simpler cellular automaton that acts on \( 2 \times 2 \) blocks of cells. In particular, this means that patterns that are initially made up of \( 2 \times 2 \) blocks will forever be made up of \( 2 \times 2 \) blocks under this evolution rule. Because of the simplicity of the emulated block cellular automaton, it has many properties in common with elementary cellular automata \([3, 4]\) and in particular emulates Wolfram’s rule 90 \([2]\).

Although the rough behaviour and statistics of 2x2 are similar to those of Life, the patterns of 2x2 have completely different structure and thus are interesting in their
own right. Furthermore, many questions that have been answered about Life remain open in 2x2, such as whether or not it contains guns or replicators. Interestingly, both of these types of patterns are known to exist in the nearby rule B368/S12578. However, this rule lacks the block cellular automaton simulation properties of B36/S125 that will be discussed later.

2x2 has a basic \( c/8 \) diagonal glider that occurs fairly commonly, though it is larger than the standard Life glider and is thus more difficult to construct. The first infinitely-growing pattern to be discovered was a wickstretcher based on the glider. This wickstretcher is displayed in Fig. 7.2 with alive cells in black and dead cells in white.

Other spaceships were found via computerized searches carried out by Alan Hensel, Dean Hickerson, David Bell, and David Eppstein in the 1990s. One of the most important spaceship discoveries was a \( c/3 \) diagonal spaceship, which showed that it is possible for spaceships to travel faster in the 2x2 universe than in the standard Life universe, despite most of the easy-to-find spaceships being quite slow. We will see that \( c/3 \) is the diagonal speed limit in 2x2, much like \( c/4 \) is the diagonal speed limit in Life. Several derived results will also apply to other Life-like cellular automata, and we will note when this is the case, although our focus and motivation will be the 2x2 rule.

### 7.2 Ash and Common Patterns

One of those most interesting aspects of 2x2 is the large variety of still lifes and oscillators that appear naturally as a result of evolving randomly-generated starting patterns (known as *soup*). Many simple still lifes are familiar from the standard Life rules, such as the tub, beehive, aircraft carrier, loaf, and pond. More commonly-occurring, however, are simple “sparse” still lifes that are not stable in Life, such