Axiomatics and self-reference
Reflections about Hermann Grassmann’s contribution to axiomatics

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The (never ending?) debate

Hermann and Robert Grassmann’s writings published between 1844 and 1891 led to a lively debate among scientists, philosophers, and historians about the contribution they bring to the advancement of axiomatics.

Gottfried Martin, Michael Otte, Hans-Niels Jahnke, Judson Webb, and Arno Zaddach\(^1\) regard some of the general fundamental methodological tenets and/or some of the mathematical techniques developed by the Grassmann brothers as major contributions to the emergence of the new axiomatic spirit which, according to common wisdom, is said to have found its accomplished expression in David Hilbert’s work.

Jean Cavaillès, Hao Wang, Jean Dieudonné, Michel Guillaume, Albert Lewis, Jean-Luc Dorier, and Hans-Joachim Petsche hold the opposite view. According to these scholars, even though many of the mathematical concepts and techniques developed by the Grassmann brothers were subsequently incorporated, for instance, in the axiomatic

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\(^1\) To these names one could also add those of Hermann von Helmholtz, Giuseppe Veronese, Hermann Hankel, Gottlob Frege, and Otto Hölder. All of them link the contributions of the Grassmann brothers (as a rule those of Hermann) to the axiomatic tradition, compare [Radu 2000a, b, 2005] and the literature quoted therein for details.
treatment of arithmetic and in the axiomatic presentation of the abstract vector-space concept, Hermann Grassmann’s approach to the foundations of mathematics is, on the whole, antithetical to axiomatics.

Some of these scholars also argue that the philosophical ideas that shaped Hermann Grassmann’s views on the foundations of mathematics, most notably the ideas of Schleiermacher’s *Dialectic*, are incompatible with modern axiomatics. The mere fact that modern axiomatic treatments of arithmetic and algebra rely heavily on mathematical techniques developed by Hermann Grassmann or the direct acknowledgment of Richard Dedekind and of Giuseppe Peano of their debt to Grassmann, are not seen as proving the presence of the modern axiomatic spirit in Grassmann’s work. Any resemblance with existing modern axiomatic theories, so it seems, is purely accidental, and in any case unintentional.

Last but not least, there is a third group of scholars who are led to consider the impact of the general methodological ideas and of the mathematical techniques developed by the Grassmann brothers on axiomatics indirectly, while discussing other subjects. Among the scholars belonging to the third group I would mention Abraham Robinson, Michael Crowe, Donald Gillies, Hans Wussing, Paul Shields, and Volker Peckhaus.

On the whole, the ideas put forward by these scholars either support more or less directly the view that the Grassmann brothers did not bring any major contributions to the emergence of the new axiomatic spirit or they contain general statements that seem compatible with an interpretation of the contributions of the Grassmann brothers as belonging to the axiomatic tradition.

A word of warning must be added to the classification attempted here. The approaches of the previously mentioned authors are very different in scope, depth, philosophical perspective, general account of axiomatics underpinning them, and so on. Grouping them together is, therefore, to a certain extent misleading. By doing this, I do not wish to suggest the existence of any wider agreement between the positions defended by the various scholars. The only agreement involved concerns the general assessment of the place of axiomatics in the work of the Grassmann brothers or the

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2 This view is held, for instance by Hao Wang in his influential paper from 1957 [Wang 1957] and by Hans-Joachim Petsche 2006 in his biography of Hermann Grassmann [Petsche 2006].

3 Some of these authors provide substantial space to the discussion of the history of axiomatics without, however, even mentioning Hermann or Robert Grassmann (see, for instance [Radu 2000b, Sect. I.2] for details). This type of approach also contributes to the general tendency to place the work of the Grassmann brothers outside the axiomatic tradition. Almost all authors mentioned in this paper, however, do discuss some of the contributions of the Grassmann brothers, even if not all explicitly address axiomatics.

4 Compare [Radu 2000b; 2005] for details.