of the local intersection numbers, can be computed quite easily for plane curves by means of other global invariants, namely the orders of C and C', because the intersection ring of $P_2(\mathbb{C})$ is so simple. Thus:

**Bézout's theorem**

The intersection number $C \cdot C'$ of two plane curves $C$ and $C'$ of orders $m$ and $m'$ is given by

$$C \cdot C' = m \cdot m'.$$

**Proof**: By Propositions 3, 6 and 7 we have:

$$C \cdot C' = <[C],[C']> = <m[L],m'[L]> = m \cdot m' <[L],[L]> = mm'.$$

7. **Some simple types of curves**

7.1 **Quadrics**

Next to lines, quadrics are the simplest plane curves. From the complex-projective standpoint they are the analogues of the conic sections of antiquity. If one also admits curves with multiple components, and thus understands the quadrics to include all "curves" with equations of degree 2, then a quadric is just a curve with a homogeneous equation

$$\sum_{i,j=0}^2 a_{ij} x_i x_j = 0,$$

where one can assume without loss of generality that $a_{ij} = a_{ji}$. The polynomial $\sum_{i,j=0}^2 a_{ij} x_i x_j$ is a form of degree 2, a quadratic form. If $A$ is the matrix $(a_{ij})$, then one can write this form in matrix fashion as follows:

$$x^t A x.$$

Under a linear coordinate transformation $x = By$, this goes over to

$$y^t B^t A By.$$

Under linear coordinate transformations, a quadric transforms like a quadratic form, and we can therefore bring the equation into a normal form by bringing the matrix $A$ into a normal form through a transformation $B^t A B$. Now it is known from linear algebra that, over the complex numbers, a symmetric matrix can always be brought into diagonal form by such a transformation, with 1's and 0's on the diagonal. With real symmetric matrices one can arrive by real transformations at

a diagonal form in which the diagonal entries are 0 or \( \pm 1 \). The projective classification of quadrics follows immediately from this.

**Theorem 1**

Relative to suitable homogeneous coordinates, the equation of each complex projective plane quadric \( Q \) has exactly one of the following three normal forms

(i) \( x_0^2 + x_1^2 + x_2^2 = 0 \)

(ii) \( x_0^2 + x_1^2 = 0 \)

(iii) \( x_0^2 = 0 \).

In case (i) \( Q \) is an irreducible, non-singular quadric, in case (ii) \( Q \) decomposes into two distinct lines, and in case (iii) \( Q \) is a double line.

Thus the complex-projective classification of quadrics is very simple: there are only three different kinds. The most interesting are, of course, the non-singular curves.

What else can one say about the non-singular quadrics? Let us go from the normal form \( x_0^2 + x_1^2 + x_2^2 = 0 \) to another normal form by introducing new coordinates \( y_0 = x_0, y_1 = ix_1 + x_2, y_2 = ix_1 - x_2 \). Then one obtains the equation

\[
y_0^2 - y_1y_2 = 0.
\]

This equation has the advantage, among others, that it immediately shows the non-singular quadric \( Q \) to be isomorphic, as an abstract curve, to the 1-dimensional complex projective space \( P_1(\mathbb{C}) \), i.e. to the Riemann number sphere. Namely, we choose homogeneous coordinates \( (z_0, z_1) \) in \( P_1(\mathbb{C}) \) and consider the mapping

\[
(z_0, z_1) \mapsto (z_0z_1^2, z_0^2, z_1^2).
\]

One sees immediately that this is an everywhere defined, rational, bijective mapping

\[
P_1(\mathbb{C}) \rightarrow Q
\]

and that the inverse mapping is likewise rational and everywhere defined. This mapping is therefore an isomorphism of \( P_1(\mathbb{C}) \) onto \( Q \), when one considers them as abstract curves. In particular, the two curves are homeomorphic as topological spaces, namely, both are homeo-