Mathematical Modelling in Normative Development Planning

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Abstract

The paper presents an approach for modelling long-term development of complex economic systems. Concerning management of a complex system one has to single out the following three main stages of research: goal formulation, calculation of planned or ideal system's development strategy (trajectory), and selection of additional controls or changes of the preliminary chosen controls which may allow one to compensate influence of certain random factors or factors which were not taken into account at the preliminary stage of analysis.

In this paper we deal with the second stage of the research, i.e., the main problem being solved is selection of those controls which can guarantee attainment of the system's goals and which is optimal with respect to a given criteria.

In order to be more specific we provide here a general problem statement in a mathematical form. Let a hypothetical economy consist of M production sectors. Each sector produces only one product. Balance equations are then as follows:

\[ Q(t+1) = Q(t) + x(t) - q(t) - C(t) - G(t) \]

\[ Q(t_0) = Q_0 \]  \hspace{1cm} (1)

where \( Q(t) \), \( x(t) \), \( q(t) \), \( C(t) \), and \( G(t) \) are stock of the production output, gross output, intermediate products, final consumption, and governmental consumption, respectively. All these variables are vectors of dimension M. In addition to production activity, a support program is formulated in each industrial sector. The
support program is a set of activities, which are directed to technological change, expansion of existing industrial capacities, conversion of industry in order to produce new goods, etc. Let the support program of sector i consist of Ni activities.

The dynamic equation for program completion is the following:

\[ z^i(t+1) = \phi_i(z^i(t), u^i(t)) , \quad z(t_0) = z_0 , \]  

where \( z^i_j(t) \) is an index characterizing the extent of the \( j^{th} \) activity completion in a year \( t \) and \( u^i_j(t) \) intensity of carrying out the \( j^{th} \) activity in the \( i^{th} \) program within a year \( t \). \( z^i \) and \( u^i \) are vectors of dimension \( Ni \). Function \( \phi_i \) defines transition of the program from one stage to another.

The performance of the program is subject to resource (in a wide sense) constraints

\[ a_i(z^i(t), u^i(t)) \leq A_i(t) , \]  

where given functions \( a_i \) and \( A_i \) define resource consumption and supply in year \( t \).

Certain constraints on a sequence of carrying out activities can be applied. We represent these constraints in a quite general form:

\[ u^i(t) \in \mathcal{U}(z^i(t)) , \]  

where \( \mathcal{U} \) domain of feasible \( u \), when a program is in the stage \( z^i(t) \). The program is completed if \( z^i(t) \geq z^i_t \). Similarly, to support programs in industry, the government formulates its general purpose program. It can be described as follows:

\[ y(t+1) = \psi(y(t), v(t)) , \quad y(t_0) = y_0 \]  

\[ b(y(t), v(t)) \leq G(t) . \]

Now we come back to the description of a production process. All the goods bought by sector i are divided into two parts: