GREEN’S FUNCTION FOR THE AHARONOV-BOHM EFFECT WITH A NON-ABELIAN GAUGE GROUP

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In the case of more than one solenoid, the computation of the Green’s function for the Aharonov-Bohm effect becomes rather complicated. In this paper, we discuss two possible approaches to this problem. We start with the Schulman’s approach based on the Feynman path integral. Applying this technique, recently having been generalized by several authors to non-Abelian gauge groups, we are able to derive some explicit formulae in the two-solenoid case. On the other hand, the interpretation of the problem in the framework of the theory of self-adjoint extensions allows us to apply the Krein’s formula. But in this case, some open problems still remain.

1. INTRODUCTION

In this paper, we consider a non-relativistic particle moving in an external and generally non-Abelian gauge field with vanishing field strength. The problem here is discussed from the mathematical point of view. More precisely, the aim is to compute the Green’s function. Provided the underlying configuration space is multiply connected, the corresponding Schrödinger operator need not be equivalent to a multiple of the Laplace-Beltrami operator. In fact, this is the meaning of the Aharonov-Bohm effect [1] on the theoretical level.
The generalization of the magnetic field to the non-Abelian gauge field was proposed by Wu and Yang [2]. One way how to attack this problem is to use the Feynman path integral. This approach was elaborated by Schulman [3] for the $U(1)$ gauge group and then it was generalized to the non-Abelian case by Sundrum and Tassie [4] and by Oh, Soo and Lai [5]. But all these deliberations, sketched here in Sec.2, can be concentrated into the unique relation which we shall refer to as the Schulman's Ansatz. The simplest example providing the truly non-Abelian effect is the two-solenoid with the doubly punctured plane as the configuration space [4,5,6]. In this concrete case, one is able to derive more explicit formulae [7] which are presented here without some details in Sec.3. On the other hand, the problem can be considered as belonging to the theory of self-adjoint extensions. The possible application of the Krein's formula is discussed in Sec.4.

2. THE SCHULMAN'S ANSATZ

Let $M$ be connected but generally multiply connected Riemannian manifold and $A_\nu$ be a gauge field with the gauge group $U(N)$ and such that the field strength $F_{\mu\nu} = d_\mu A_\nu - d_\nu A_\mu + [A_\mu, A_\nu]$ vanishes on $M$. It can be shown that, up to equivalence, all such gauge fields, i.e., all flat connections on $M$ are in one-to-one correspondence with unitary representations $U$ of the fundamental group $\Gamma = \pi_1(M, x_{\text{ref}})$. Provided a reference point $x_{\text{ref}} \in M$ is fixed, the unitary representation is given by the parallel transport. The inverse procedure is also clear. Let $\tilde{M}$ designate the universal covering space which is again a Riemannian manifold. We accept the formalism when $\Gamma$ acts on $\tilde{M}$ from the left and the group multiplication in $\Gamma$ is defined as follows: $[\gamma_1][\gamma_2] = [\gamma_1 \circ \gamma_2]$, where $\gamma_1 \circ \gamma_2$ means that the curve $\gamma_2$ follows the curve $\gamma_1$. Then the quotient $\Gamma \backslash \tilde{M}$ coincides with $M$ and $\tilde{M}$ is a principal bundle over $M$ with the structure group $\Gamma$. Since $\dim \tilde{M} = \dim M$, in this principal bundle there exists only one connection and it is necessarily