Alpine Downhill and Speed-Skiing

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1 Introduction

The equations given in the previous chapter for a downhill cross-country skier are also applicable to an alpine downhill skier. However, we have ignored the lift force on the cross-country skier since the velocity is relatively low. For an alpine skier and especially a speed-skier the aerodynamic lift on the body will be important. Since the lift will indirectly reduce the ski friction, a reduction of the friction coefficient $\mu$ could be applied. For the speed-skier at velocities around 250 km/h, the lift force will seriously influence the stability of the athlete.

The next paragraph will introduce the Brachistochrone problem which describes the geometry of the downhill slope for the fastest descend when friction and aerodynamic drag is ignored. This also means that for given downhill course a large initial velocity of the skier (or e.g. of a bobsleigh) is advantageous.

2 The Brachistochrone problem

The great mathematicians of the 17th Century (Newton, Leibniz and Bernoulli among others) were engaged in finding the two-dimensional curve for a body which should move from point $A$ to a lower point $B$ under the influence of gravity and with the shortest time (Brachistochrone in Greek). Ignoring friction in the problem, the solution is related to the cycloid path (see Figure 1) which is given in a parametric form as:

\[ X/R = (2\pi/360)\varphi - \sin\varphi \]

\[ Y/R = 1 - \cos\varphi \]
Figure 1. The generation of a cycloid (Illustration: Trondheim Science Centre).

Here the parameter $\varphi$ [deg] defines the cycloid in a Cartesian space $X,Y$ and $R$ is the radius of the rolling circle. If we invert the cycloid about the horizontal $X$-axis we will obtain the inverted cycloid (or the Brachistochrone) as shown in Figure 2.

The time and length given below are in dimensionless form with the inverted half-cycloid representing the reference path, see Table 1. The arc of the cycloid will always be the Brachistochrone when compared to the straignt lines from $A$ to $B_1$, $B_2$ or $B_3$. It is interesting to note that even when the straight line is shorter than the inverted half-cycloid, the time it takes to descend on a straight line is 18.5 % longer.