Variational Formulations of Interior Structural-Acoustic Vibration Problems

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Abstract It is proposed to present appropriate variational formulations for linear vibration of elastic structure coupled with an internal acoustic fluid. Hybrid passive/active damping treatments will be investigated for noise and vibration reduction problems.

1 Introduction

It is proposed to present appropriate variational formulations for linear vibration of elastic structure coupled with an internal inviscid, homogeneous, compressible fluid (liquid or gas), gravity effects being discarded in the presence of a free surface. Hybrid passive/active damping treatments will be investigated for noise and vibration reduction problems. It should be noted that generally, active structural treatments (using for example piezoelectric smart materials) are effective in the low frequency range, while passive structural treatments (such as viscoelastic materials, porous insulation...) are effective for higher frequency domain.

In all the analyzed variational formulations, the structure will be described by a displacement field (the piezoelectric structure being described by an additional electric potential field). Concerning the fluid, instead of a description through a displacement field (for which we refer for example to (Bermúdez and Rodríguez, 1994; Park et al., 2001)) we will choose a scalar description through a pressure and/or a displacement potential field (Morand and Ohayon, 1995; Ohayon, 2004a,b).

Dissipative behavior is introduced through a fluid-structure wall damping modeling by local impedance connected with a viscoelastic Kelvin-Voigt type of constitutive equation. When taking into account dissipative structural-acoustic behavior through a local impedance constitutive equation, the problem becomes strongly frequency dependent (Kehr-Candille and Ohayon, 1992). In this presentation, we will use a simplify but rather
general constitutive model of Kelvin-Voigt type through the introduction of a scalar interface variable which allows the problem to be reduced to a classical vibration damping problem (Deü et al., 2006; Larbi et al., 2006). This impedance model, though local, may represent relatively satisfactory porous medium (on a rigorous manner, a precise three-dimensional description of the porous medium at the fluid-structure interface would be necessary through Biot type approach (Davidsson and Sandberg, 2006)).

For piezoelectric structures (active treatments), structural-acoustic conservative formulation are extended in order to take into account electromechanical contributions. Here also, appropriate choice of variables has been investigated and leads to the introduction of the electric potential as an additional variable (Deü et al., 2008; Larbi et al., 2007).

For all the formulations, finite element discretization is discussed. Numerical results are then presented in order to illustrate the accuracy and versatility of the methodologies.

2 Conservative Structural-Acoustic Coupled Problem

Let us considered the free vibrations of an elastic structure completely filled with a homogenous, inviscid and compressible fluid, neglecting gravity effects. We establish in this section the variational formulation of the spectral problem and the corresponding matrix equations.

2.1 Local Equations

We consider an elastic structure occupying the domain $\Omega_S$ at the equilibrium. The structure is subjected to a prescribed displacement $u^d$ on a part $\Gamma_u$ and to surface force density $F^d$ on the complementary part $\Gamma_\sigma$ of its external boundary. The interior fluid domain is denoted by $\Omega_F$ and the fluid-structure interface by $\Sigma$ (see Figure 1).

The structure is supposed to be relevant of classical linearized elasticity theory. Therefore, the stress tensor $\sigma_{ij}$ is related to the linearized deformation tensor $\varepsilon_{ij}$ by the constitutive law

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$$  \hspace{1cm} (1)

where $c_{ijkl}$ are the coefficients of elasticity. We denote by $\rho_S$ the mass density of the structure and $n^S_i$ the unit normal external to $\Omega_S$.

Since the compressible fluid is assumed to be inviscid, instead of describing its motion by a fluid displacement vector field, which requires an appropriate discretization of the fluid irrotationality constraint (Bermúdez and Rodríguez, 1994), we use the pressure scalar field $p$. Let us denote by