Special functions is a set of some classes of particular functions that have attractive or useful properties arising from solutions of theoretical and applied problems in different areas of mathematics.

Special functions can be defined by means of power and trigonometric series, series of orthogonal functions, infinite products, generating and distribution functions, integral representations, sequential differentiation, transcendental, differential, difference, integral, and functional equations.

Maple includes over 200 special functions. We will consider some the most important special functions (see \texttt{?inifcn}, \texttt{?index[package]}, \texttt{?index[function]}, \texttt{?FunctionAdvisor}).

Mathematica includes all the common special functions of mathematical physics found in standard handbooks. It should be noted that the definitions (including normalizations and special values) of any particular special function can be different in handbooks and also in Maple and Mathematica (e.g., the Mathieu functions). We will discuss some the most important special functions.

9.1 Functions Defined by Integrals

Gamma, Beta, digamma, and polygamma functions

Maple: \texttt{GAMMA}, \texttt{Beta}, \texttt{Psi(x)}, \texttt{Psi(n,x)}.

\begin{verbatim}
plot(GAMMA(x),x=-5..5,-20..20,numpoints=500,color=blue);
GAMMA(z+2); convert(GAMMA(z+2), factorial);
\end{verbatim}
convert(z!, GAMMA); simplify(z*GAMMA(z)); GAMMA(1/2); GAMMA(-1/2); GAMMA(5/2); Beta(2,9); expand(Psi(73)); evalf(expand(Psi(2,73))); evalf(Psi(2,73));

Mathematica: Gamma, Beta, PolyGamma[x], PolyGamma[n,x].

Plot[Gamma[x],{x,-5,5},PlotRange->{-20,20},PlotStyle->Hue[0.9]]
{z*Gamma[z]//FunctionExpand, Gamma[1-2*I]//N,
 D[Gamma[z],z], Gamma[1/2], Gamma[-1/2], Gamma[5/2]}
{Beta[2,9], N[{Erf[1],Erfc[1]}]}
{PolyGamma[73], PolyGamma[2,73]//FunctionExpand,
 PolyGamma[2,73]//N}

Exponential, logarithmic, polylogarithmic, and trigonometric integrals

Maple: Ei, Li, dilog, polylog, Si, Ci.

expand(Ei(5,x)); evalf(Si(5)); convert(Li(x), Ei);
convert(Ci(x),Ei); dilog(-1/2); evalf(polylog(2,-1/2));
evalf(convert(series(Li(x),x=2,9),polynom));
diff(Ci(x),x); limit(int(Si(x),x),x=0);

Mathematica: ExpIntegralE, LogIntegral, PolyLog,
SinIntegral, CosIntegral.

{ExpIntegralE[5,x]//FunctionExpand, SinIntegral[5]//N}
Series[LogIntegral[x],{x,2,9}]//Normal//N
{PolyLog[2,-1/2]//N, D[CosIntegral[x],x],
Limit[Integrate[SinIntegral[x],x],x->0]}

Error functions, Fresnel integrals

Maple: erf, erfc, FresnelC, FresnelS.

evalf(erf(1)); evalf(erfc(1));
plot([FresnelC(z),FresnelS(z),z=-10..10],-1..1,-1..1,
 scaling=constrained,color=blue,axes=boxed);