10 Sensitivity Analysis: Generalized Coordinate Kinematic Systems

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We now apply the afore developed analysis and sensitivity analysis to kinematically driven rigid body mechanisms. Initially we perform a position analysis, i.e. we determine the mechanism configuration for a given time. This analysis parallels the nonlinear static finite element analysis of Section 9.2. Through differentiation we then perform velocity and acceleration analyses. This procedure is akin to our sensitivity analysis if we view time $t$ as the parameter $d_i$ of interest. Next we evaluate reaction forces in an inverse dynamic analysis. Such forces are often used in subsequent finite element analyses to determine the stress distribution in the mechanism’s components. And finally we perform a sensitivity analysis in its own right to determine how the generalized position, velocity, acceleration and generalized reaction force vectors change as we perturb a model parameter $d_i$, e.g. a link dimension.

10.1 Position Analysis

Each body in the mechanism is modeled as a rigid body and hence each body has 3(6) degrees-of-freedom in a two(three)-dimensional analysis. In the position analysis we are concerned with evaluating these degrees-of-freedom.

We can think of the bodies as the nodes in the spring system. Here however, the “nodes” have multiple degrees-of-freedom. For our example, and without loss of generality, we consider two-dimensional analyses and hence each body $i$ has 3 degrees-of-freedom: the two components of the body $i$ mass center position vector $r_i$ and its orientation $\phi_i$ of the body $\alpha x_i y_i$ coordinate system with respect to the global coordinate system $OXY$, cf. Figure 10.1. Without loss of generality, we require origin of the body $i$ coordinate system to coincide with the body $i$ mass center. These body degrees-of-freedom comprise the body $i$ generalized coordinate vector $q_i = [ r_i^T \quad \phi_i ]^T$ and the collection of the body generalized coordinate vectors form the mechanism generalized coordinate vector $q = [ q_1^T \quad q_2^T \quad \cdots ]^T$. Again, this is done in the same way that the node degrees-of-freedom $U_i$ are assembled to form the node displacement vector $U_{ur}$.

Once we know the generalized coordinates, we can determine the location of any point on any body in the mechanism. As seen in Figure 10.1 we can express
the position vector $r^{P_i}$ with respect to the global coordinate system $OXY$ for any point $P^i$ that moves with body $i$ as

$$r^{P_i} = r_i + R(\phi_i) r^{P_i}_i$$  \hspace{1cm} (10.1)

where $r^{P_i}_i$ is the point $P^i$ position vector with respect to the body $i$ coordinate system and

$$R(\phi) = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$  \hspace{1cm} (10.2)

is a rotation matrix. Note that $r^{P_i}_i$ is constant.

The mechanism is formed by connecting the bodies with a series of joints. For example, in Figure 10.1 a revolute joint $l$ is used to connect bodies $i$ and $j$. We