6 Independent Variable Formulations

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6.1. Introduction

In Chapter 5 the dependent variable formulations for simulation of multibody systems were described. The derivation of equations of motion in terms of dependent states, though conceptually simple and easy to handle, leads to large-dimension governing DAEs, which results in computationally inefficient algorithms, burdened further with the constraint violation problem. An old and legitimate approach to the dynamics formulation is therefore to use a minimal number of independent state variables for a unique representation of motion by means of pure reduced-dimension ODEs. The numerical integration of the ODEs is usually far more efficient compared to the integration of the governing DAEs, and the numerical solution is released from the problem of constraint violation. On the other hand, the minimal-form ODE formulations require often more modeling effort and skill. Effective and computer-oriented modeling procedures of this type are thus highly desirable.

In this chapter some basic reduced-dimension ODE formulations used for simulation of multibody systems are reviewed. Firstly, the joint coordinate method for open-loop systems is described using the geometrical concepts of the projection method. The provided matrix formulation of the arising motion equations is supplemented with a compact scheme for the determination of joint reactions. A velocity partitioning method for obtaining the minimal-dimension dynamic equations is then reported, followed by a general projective scheme for independent variable formulations. A relevance of the projective scheme to Gibbs’-Appell’s equations and Kane’s equations is shown. The reduced-dimension ODE formulations for closed-loop systems are finally discussed and illustrated through examples.

6.2. Joint Coordinate Formulation for Open-Loop Systems

6.2.1. Joint Coordinates

For an open-loop multibody system, that is a system with tree graph structure, the joint coordinates \( q = [q_1, \cdots, q_k]^T \) are most natural independent coordinates that describe the system position on its configuration manifold \( \mathcal{K} \) (see Section 4.3). An individual coordinate \( q_i \) is related to a particular joint, and describes the relative
configurations of the adjacent bodies. There are thus one or more joint coordinates in each joint, which can be either rotational or translational, equal in number to the number of relative degrees of freedom. The vector $\mathbf{q}$ for a multibody system contains all the joint coordinates (and the absolute coordinates of the base body if it is not the ground), and its dimension is equal to the number $k$ of degrees of freedom of the system.

![Figure 6.1. Joint coordinates: a) relative, b) absolute.](image)

Joint coordinates describe, by definition, relative motions of the adjacent bodies in the joints, often referenced to as \textit{relative joint coordinates}, illustrated in Figure 6.1a. For planar systems, and related to revolute joints (rotational coordinates), it is judicious to use \textit{absolute joint coordinates} as well, seen in Figure 6.1b. The use of absolute joint coordinates is usually simpler and leads to more concise dynamics formulations. Evidently, using the procedures of Section 4.2.2, the motion equations can effectively be transformed from one set of joint coordinates to the other.

The derivation of equations of motion in joint coordinates is based on explicit relationship relating $n$ absolute coordinates $\mathbf{y}$ of the composite bodies and $k$ joint coordinates $\mathbf{q}$, $\mathbf{y} = \mathbf{g}(\mathbf{q})$, which is always attainable for open-loop systems. As motivated in Section 4.3, and with reference to Equation (4.15), the above relationship expresses the explicit constraint equations due to the kinematic joints in the system. The $m$ ($k = n - m$) constraint equations given implicitly in $\mathbf{y}$, $\mathbf{\Phi}(\mathbf{y}) = \mathbf{0}$, are then satisfied identically when expressed in the joint coordinates, $\mathbf{\Phi}[\mathbf{g}(\mathbf{q})] = \mathbf{0}$. The relationship $\mathbf{y} = \mathbf{g}(\mathbf{q})$ leads then to the velocity and acceleration transformation formulae (4.16) and (4.17), which are related to the implicit constraint equations (4.13) and (4.14), respectively, at the velocity and acceleration levels, through relations (4.18) and (4.19).