Selecting the Best Location for a Meteorological Tower: A Case Study of Multi-objective Constraint Optimization

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Abstract. Using the problem of selecting the best location for a meteorological tower as an example, we show that in multi-objective optimization under constraints, the traditional weighted average approach is often inadequate. We also show that natural invariance requirements lead to a more adequate approach – a generalization of Nash's bargaining solution.

Case Study. We want to select the best location of a sophisticated multi-sensor meteorological tower. We have several criteria to satisfy.

For example, the station should not be located too close to a road, so that the gas flux generated by the cars do not influence our measurements of atmospheric fluxes; in other words, the distance x_1 to the road should be larger than a certain threshold t_1: $x_1 > t_1$, or $y_1 \stackrel{\text{def}}{=} x_1 - t_1 > 0$.

Also, the inclination x_2 at the should be smaller than a corresponding threshold t_2, because otherwise, the flux will be mostly determined by this inclination and will not be reflective of the atmospheric processes: $x_2 < t_2$, or $y_2 \stackrel{\text{def}}{=} t_2 - x_2 > 0$.

General Case. In general, we have several such differences $y_1, \ldots, y_n$ all of which have to be non-negative. For each of the differences y_i, the larger its value, the better.

Multi-criteria Optimization. Our problem is a typical setting for *multi-criteria optimization*; see, e.g., [1, 4, 5].

Weighted Average. A most widely used approach to multi-criteria optimization is *weighted average*, where we assign weights $w_1, \ldots, w_n > 0$ to different criteria y_i and select an alternative for which the weighted average $w_1 \cdot y_1 + \ldots + w_n \cdot y_n$ attains the largest possible value.

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Additional Requirement. In our problem, we have an additional requirement – that all the values y_i must be positive. Thus, we must only compare solutions with $y_i > 0$ when selecting an alternative with the largest possible value of the weighted average.

Limitations of the Weighted Average Approach. In general, the weighted average approach often leads to reasonable solutions of the multi-criteria optimization problem. However, as we will show, in the presence of the additional positivity requirement, the weighted average approach is not fully satisfactory.

A Practical Multi-criteria Optimization Must Take into Account That Measurements Are Not Absolutely Accurate. Indeed, the values y_i come from measurements, and measurements are never absolutely accurate. The results $\widetilde{y}_i$ of the measurements are close to the actual (unknown) values y_i of the measured quantities, but they are not exactly equal to these values. If

- we measure the values y_i with higher and higher accuracy and,
- based on the resulting measurement results $\widetilde{y}_i$, we conclude that the alternative $y = (y_1, \ldots, y_n)$ is better than some other alternative $y' = (y'_1, \ldots, y'_n)$,

then we expect that the actual alternative y is indeed either better than y' or at least of the same quality as y'. Otherwise, if we do not make this assumption, we will not be able to make any meaningful conclusions based on real-life (approximate) measurements.

The Above Natural Requirement Is Not Always Satisfied for Weighted Average. Let us show that for the weighted average, this "continuity" requirement is not satisfied even in the simplest case when we have only two criteria y_1 and y_2. Indeed, let $w_1 > 0$ and $w_2 > 0$ be the weights corresponding to these two criteria. Then, the resulting strict preference relation $\succ$ has the following properties:

- if $y_1 > 0$, $y_2 > 0$, $y'_1 > 0$, and $y'_2 > 0$, and $w_1 \cdot y_1 + w_2 \cdot y_2 > w_1 \cdot y'_1 + w_2 \cdot y'_2$, then
$$y = (y_1, y_2) \succ y' = (y'_1, y'_2);$$
- if $y_1 > 0$, $y_2 > 0$, and at least one of the values y'_1 and y'_2 is non-positive, then
$$y = (y_1, y_2) \succ y' = (y'_1, y'_2).$$

Let us consider, for every $\varepsilon > 0$, the tuple $y(\varepsilon) \stackrel{\text{def}}{=} \left(\varepsilon, 1 + \dfrac{w_1}{w_2}\right)$, with $y_1(\varepsilon) = \varepsilon$ and $y_2(\varepsilon) = 1 + \dfrac{w_1}{w_2}$, and also the comparison tuple $y' = (1, 1)$. In this case, for every $\varepsilon > 0$, we have

$$w_1 \cdot y_1(\varepsilon) + w_2 \cdot y_2(\varepsilon) = w_1 \cdot \varepsilon + w_2 + w_2 \cdot \frac{w_1}{w_2} = w_1 \cdot (1 + \varepsilon) + w_2$$

and

$$w_1 \cdot y'_1 + w_2 \cdot y'_2 = w_1 + w_2,$$