Chapter 1. Why Study Scientific Computing?

The emergence of scientific computing as a vital part of science and engineering coincides with the explosion in computing power in the past 50 years. Many physical phenomena have been well understood and have accurate models describing them since the late 1800s, but before the widespread use of computers, scientists and engineers were forced to make many simplifying assumptions in the models in order to make them solvable by pencil-and-paper methods, such as series expansion. With the increase of computing power, however, one can afford to use numerical methods that are computationally intensive but that can tackle the full models without the need to simplify them. Nonetheless, every method has its limitations, and one must understand how they work in order to use them correctly.

1.1 Example: Designing a Suspension Bridge

To get an idea of the kinds of numerical methods that are used in engineering problems, let us consider the design of a simple suspension bridge. The bridge consists of a pair of ropes fastened on both sides of the gorge, see Figure 1.1. Wooden supports going across the bridge are attached to the ropes at regularly spaced intervals. Wooden boards are then fastened between the supports to form the deck. We would like to calculate the shape of the bridge as well as the tension in the rope supporting it.

1.1.1 Constructing a Model

Let us construct a simple one-dimensional model of the bridge structure by assuming that the bridge does not rock side to side. To calculate the shape of the bridge, we need to know the forces that are exerted on the ropes by the supports. Let $L$ be the length of the bridge and $x$ be the distance from one
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Figure 1.1. A simple suspension bridge.

Figure 1.2. Force diagram for the bridge example.

end of the bridge. Assume that the supports are located at \( x_i, i = 1, \ldots, n \), with \( h \) being the spacing between supports. Let \( w(x) \) be the force per unit distance exerted on the deck at \( x \) by gravity, due to the weight of the deck and of the people on it. If we assume that any weight on the segment \([x_{i-1}, x_i]\) are exerted entirely on the supports at \( x_{i-1} \) and \( x_i \), then the force \( f_i \) exerted on the rope by the support at \( x_i \) can be written as

\[
 f_i = \left( \int_{x_{i-1}}^{x_i} w(x)(x - x_{i-1}) \, dx + \int_{x_i}^{x_{i+1}} w(x)(x_{i+1} - x) \, dx \right). \tag{1.1}
\]

We now consider the rope as an elastic string, which is stretched by the force exerted by the wooden supports. Let \( u_i \) be the height of the bridge at \( x_i \), \( T_{i-1/2} \) be the tension of the segment of the rope between \( x_{i-1} \) and \( x_i \), and \( \theta_{i-1/2} \) be the angle it makes with the horizontal. Figure 1.2 shows the force diagram on the rope at \( x_i \).

Since there is no horizontal displacement in the bridge, the horizontal forces must balance out, meaning

\[
 T_{i-1/2} \cos(\theta_{i-1/2}) = T_{i+1/2} \cos(\theta_{i+1/2}) = C,
\]