Chapter 7
Representing Thick Indifference in Spatial Models

Abstract. This chapter demonstrates that a fuzzy approach to modeling thick indifference can accommodate highly irregularly shaped indifference curves, even those that are concave or multi-modal. Moreover, it permits the calculation of a majority rule maximal set with relative ease under assumptions of non-separability. This approach relies on a homomorphism that permits a region of interest to be mapped to a simpler region with a suitable and natural partial ordering where the results are determined and then faithfully transferred back to the original region of interest.

7.1 Stability and Thick Indifference in Individual Preferences

It has been long known that the probability of a majority rule maximal set increases in spatial models when actors possess thick indifference over individual preferences (Bräuninger, 2007; Balke et al., 2006; Barberà and Ehlers, 2011; Gehrlein and Valognes, 2001; Skog, 1994; Sloss, 1973; Tovey, 1991). Many of the studies in this genre make use of Tovey’s (1991) concept of an epsilon-core ($\varepsilon$-core), a threshold distance in Euclidean space that must be exceeded before players distinguish between alternatives (Bräuninger, 2007; Koehler, 2001). Unless an alternative lies outside of the region defined by the $\varepsilon$-core, a player is indifferent between it and the core’s center. Essentially, actors have thick indifference curves Sloss (1973). Unfortunately, applying the approach in empirical analyses is hampered by the complexity of calculating the existence of a majority rule maximal set. It is even more problematic when thick indifference introduces irregularly shaped preference curves.

This chapter demonstrates that a fuzzy approach to modeling thick indifference can accommodate highly irregularly shaped indifference curves, even those that are concave or multi-modal. Moreover, it permits the calculation of a majority rule maximal set with relative ease under assumptions of non-separability. Section 7.2 develops the approach, which relies on a homomorphism that permits a region of interest (spatial model) to be mapped to a simpler region with a suitable and natural partial ordering where the results are determined and then faithfully transferred back to the original region of interest. Section 7.3 provides an empirical application of the approach. Section 7.4 then presents a proof of the homomorphism. Section 7.5 presents
a formal that in all but a limited number of cases, spatial models of individuals with thick indifference curves result in an empty majority rule maximal set if and only if the Pareto set contains a union of cycles. The section also completely characterizes the elements that constitute the exception for a three-person game based on the general definition for $n$ players. The substantive interpretation is that if the degree to which a majority find a given alternative acceptable is relatively high, then a stable outcome is assured under majority rule. Section 7.6 concludes with a consideration of the theoretical implications of the approach and observations on its utility for empirical studies.

7.2 Modeling Thick Indifference in Individual Preferences

The conventional approach to fuzzy spatial modeling, where $X$ is the set of alternatives, makes use of fuzzy preference relations Bezdek et al. (1978, 1979); Blin (1974); Kacprzyk and Fedrizzi (1988); Kacprzyk et al. (1992); Nurmi (1981a); Orlovsky (1978). Arguing that most data available to social sciences do not measure preference relations, Clark, Larson, Mordeson, Potter, and Wierman (2008) follow the lead of Nurmi (1981b) and use fuzzy sets to denote individual preferences. We gave consideration to individual preferences in Chapter 3.

Let $N$ be the set of political actors and $A$ be the set of alternatives. We assume that $A$ is a subset of an arbitrary universe of interest. Applied to spatial models, $A \subseteq \mathbb{R}^k$, where $\mathbb{R}$ is the set of real numbers and $k$ is the number of dimensions in Euclidean space. Let a function, $\sigma_i$, indicate the degree to which political actor $i \in N$ views a particular alternative in the policy space as more or less ideal. Thus, $\sigma_i$ is a function mapping $A$ onto the closed interval $[0, 1]$, where $\sigma_i(x) = 1$ represents all ideal policies and $\sigma_i(x) = 0$ represents all policies that are totally unacceptable to player $i$. If $\sigma_i$ is restricted to a discrete set, actors possess thick indifference. For example, $\text{Im}(\sigma_i) \subseteq T = \{0, .25, .5, .75, 1\}$ would impose preferences similar to a Likert scale, where $\text{Im}(\sigma_i)$ denotes the image of $\sigma_i$. $T$ denotes the granularity of individual preferences, how discerning players are over alternatives. We can set $T$ to any finite scale. In essence, political actors partition $A$ into a finite number of classes, each class comprising an indifference set. While the boundaries between each indifference set may be rather sharp, this problem can be resolved by increasing the granularity in the region of a boundary. Doing so does not effect our results. For ease of presentation and without loss of generalization, we consider coarse granularity at the boundaries of indifference sets in our examples.

Both the geometry representing spatial preferences and its corresponding relation space can be mapped into a simpler, more appropriate set $U$. We assume that $U$ is an arbitrary set with a partial order, making it a lattice and allowing for a simpler analysis of the relation space. Nonetheless, we can specify $U$ to be more intuitive. We specify $U = T^n$, where $T^n = \{(a_1, \ldots, a_n) \mid a_i \in T, i = 1, \ldots, n\}$ and $n = |N|$. The mapping of the relation space into $T^n$ permits the characterization of any policy space in its entirety with $n$-tuples, $(a_1, a_2, \ldots, a_n)$, which represent the specific $\sigma$-values of a policy space. Essentially, $T^n$ is a lattice of $n$-tuples with entries from $T$ under this construction.