Temporal Multivariate Networks

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In previous chapters, this book has primarily concerned itself with visualization methods for static, multivariate graphs. In a static scenario, the network has a number of attributes associated with its elements. These attribute values remain fixed and the challenge is to visualize the interactions between the network(s) and these attributes. Static multivariate graphs could be viewed as graphs with an associated high dimensional data set linked to its elements.

Time is simply another dimension in this multivariate data set that can interact with the vertices, edges, and attribute values of the network. However, humans perceive time differently as we know from our everyday interactions with the physical world. Thus, intuitively, this dimension is often handled differently when supporting the presentation of data that changes over time. Visualization applications and techniques have, and probably should, continue to exploit this fact, allowing for effective visualization methods of temporal multivariate graphs.

In this chapter, we define, characterize, and summarize the data and visualization techniques relating to temporal multivariate networks. Section 2.1.1 provides definitions and examples that characterize the networks we address in this chapter. We further refine our definitions of time in Sect. 8.2. In Sect. 8.3 we survey representations for dynamic multivariate networks and provide a survey of visualization techniques. We describe the visualization of temporal multivariate networks in the domain of software engineering in Sect. 8.4. Finally, Sect. 8.5 describes open problems in this area.

8.1 Definitions

In a variety of applications, time varying multivariate data can be viewed as evolving information networks whose structure is derived from data attributes (i.e., via similarity measures), is a specified a priory (i.e., the flow of information over an underlying network), or is the result of tracking behavioral statistics (i.e., network traces). The network and attributes can be:
inherent to the fundamental data elements that are taken to be the network vertices (name, age, gender, income, profession, interests, \ldots)

- indicators of the type of relation between the network vertices (professor of, father of, boss of, colleague of, \ldots)

- attribute derived data (time varying computational mappings from vertex attributes to edge attributes such as “pairs of stocks in markets whose performance has been above a given threshold during a time period”)

- structural derived statistics (vertex ranks, network centrality, clustering measures, \ldots)

- specified contexts in which the data occurs (Tweets related to a given set of key words for a specified time period)

In the next subsection, we adapt a model used in software engineering for the purposes of characterizing the types of dynamic, multivariate networks that can be visualized. Then, we propose mathematical formalizations of time varying multivariate networks.

### 8.1.1 Structure, Behavior, and Evolution

In a static multivariate network analysis scenario, we have a network structure, consisting of vertices and edges, as well as attributes associated with these vertices and edges. In a time varying scenario, both the graph structure and attribute values can evolve over time. In most cases, we can assume that the network structure at a given moment in time can influence how the attribute values evolve and vice versa. These interactions are in some respects very similar to those considered in some software engineering contexts [28].

Thus, we examine time varying multivariate networks appearing in biology and social networks under the lenses of structure, behavior and evolution.

- **Structure**: Pairings between elements of a complex system. Structure mostly relates to the topology of the underlying network at a given time $t$.

- **Behavior**: Observable activity. Action or reaction of system elements under a given set of stimuli. Behavior mostly refers to the attributes associated with the underlying network elements and how they change over time.

- **Evolution**: Gradual development of a configuration or pattern over time. Evolution mostly relates to the structural changes of the overall underlying network over time.

To illustrate these concepts, we provide examples in Table 8.1 drawn from the application areas considered in this book: biology, software engineering, and social networks. As an analogy to understand the overarching idea, consider a physical space, such as a building. The structure of the building is the construction at a given time. Its behavior is how people use the building and its rooms or interact with the physical structure. Its evolution may involve bringing in a construction crew to knock down walls and build new ones,