The Complexity Analysis of Chaotic Systems

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Abstract. The complexity of the sequence is an important index of quantify the performance of chaotic sequence. In order to select a higher complexity of chaotic sequence and apply it in hardware encryption system, this paper analyzes chaotic complexity quantitative analysis methods and presents the approximate entropy and permutation entropy as criterion of measuring the complexity of the chaotic sequences. Set tent, logistic and henon three kinds of chaotic systems as examples, and we analysis and comparison their complexity. It is proved that the two kinds algorithms are effective, and can distinguish different complex chaos and chaotic sequences. Researches show that the complexity of the Logistic map is greater than that of other chaotic systems. The results of the study provide the theoretical and experimental basis for the application of chaotic sequence in hardware encryption system and the information security communication.

Keywords: chaos, complexity, approximate entropy, permutation entropy.

1 Introduction

Chaos, as a classical complex phenomenon of nonlinear dynamic system, has attracted widespread attention for its broadband, noise-like, and sensitive features for initial state. In recent years, with more research on chaos, chaos has replaced the traditional pseudo random sequence in the high density of the commercial and most spread spectrum communication system [1,2].

The complexity of the sequence is not only a similarity degree of measurement between chaotic pseudo-random sequence and random sequence, but also a complexity degree of measurement by using part of the sequence to recovery
the whole. The bigger complexity of the sequence is, the smaller the possibility of recovery is. Therefore, the complexity of the sequence is an important index of quantify the performance of chaotic sequence. The researches of complexity have been attentioned by domestic and foreign scholars. Kolmogrov (1958) [3] defined a measure entropy and used it to measure the disordered degree of system movement. And then Lempel [4] et al. realized the measure entropy method by computer. Pincus (1991) [5] proposed the definition of approximate entropy through measuring the complexity of time series, and then Bandt [6] et al. proposed permutation entropy for measuring time series. Xiao Fang-hong [7] et al. proposed to apply a symbolic dynamics approach for the complexity analysis of chaotic pseudo-random sequences in 2004. Next year Larrondo [8] et al. proposed a intensive statistical complexity measure to quantify the performance of chaotic pseudorandom number generators. Chen Xiao-jun (2011) [9] et al. proposed a new complexity metric to evaluate the unpredictability of the chaotic pseudorandom sequences based on the Fuzzy Entropy.

Kolmogorov-sinai entropy proposed by Kolmogrov can measure the complexity of chaotic system, but it needs a lot of sample space and heavy computation. The symbolic dynamics approach can reduce the degree of dependence on the parameters, but before we measure the complexity, we must get the size of symbol space of the initial sequences, which is very difficult for us to obtain the priori knowledge in practice.

The article assesses randomness of Logistic, Tent and Henon mappings via approximate entropy and permutation entropy to find the better chaotic mapping, and then provides powerful basis for realization the chaotic encryption system by the hardware and the application of chaotic systems in cryptography and secure communication.

2 The Qualitative Characteristics of Chaos

Chaos, as one of the nonlinear dynamic systems has the geometry and statistical features that deterministic movement usually do not have, such as local instability while overall stability, strange attractor, continuous power spectrum, positive Lyapunov index, fractal dimension, positive measure entropy and so on. To sum up, the chaos has the following three main qualitative characteristics [10]:

1) Inherent randomness: From deterministic nonlinear system evolution process, they show random uncertainty behaviors in the chaotic sector. However, this kind of uncertainty is not from external environment random factors on the influence of the system movement, but from system of spontaneous. Another meaning of inherent randomness is local instability. Most of the chaotic system has the inherent instability and overall stability. The difference between chaotic and orderly state is stable overall while instable locality. The so-called local instability refers to the behavior for certain aspects of the system movement, which strongly depends on the initial conditions of the system.

2) Fractal dimension characteristics: Chaos has fractal dimension properties, its fractal dimension is not used to describe system geometrical shape, but to