American mathematician Paul Halmos (1916–2006), who in 1942 published the first modern linear algebra book. The title of Halmos’s book was the same as the title of this chapter.

**Finite-Dimensional Vector Spaces**

Let’s review our standing assumptions:

### 2.1 Notation \( F, V \)

- \( F \) denotes \( \mathbb{R} \) or \( \mathbb{C} \).
- \( V \) denotes a vector space over \( F \).

In the last chapter we learned about vector spaces. Linear algebra focuses not on arbitrary vector spaces, but on finite-dimensional vector spaces, which we introduce in this chapter.

**LEARNING OBJECTIVES FOR THIS CHAPTER**

- span
- linear independence
- bases
- dimension

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2. A Span and Linear Independence

We have been writing lists of numbers surrounded by parentheses, and we will continue to do so for elements of \( \mathbb{F}^n \); for example, \((2, -7, 8) \in \mathbb{F}^3\). However, now we need to consider lists of vectors (which may be elements of \( \mathbb{F}^n \) or of other vector spaces). To avoid confusion, we will usually write lists of vectors without surrounding parentheses. For example, \((4, 1, 6), (9, 5, 7)\) is a list of length 2 of vectors in \( \mathbb{R}^3 \).

2.2 Definition linear combination

A linear combination of a list \( v_1, \ldots, v_m \) of vectors in \( V \) is a vector of the form

\[
a_1 v_1 + \cdots + a_m v_m,
\]

where \( a_1, \ldots, a_m \in \mathbb{F} \).

2.4 Example In \( \mathbb{F}^3 \),

- \((17, -4, 2)\) is a linear combination of \((2, 1, -3), (1, -2, 4)\) because

\[
17 = 6(2, 1, -3) + 5(1, -2, 4).
\]

- \((17, -4, 5)\) is not a linear combination of \((2, 1, -3), (1, -2, 4)\) because there do not exist numbers \( a_1, a_2 \in \mathbb{F} \) such that

\[
17 = a_1(2, 1, -3) + a_2(1, -2, 4).
\]

In other words, the system of equations

\[
\begin{align*}
17 &= 2a_1 + a_2 \\
-4 &= a_1 - 2a_2 \\
5 &= -3a_1 + 4a_2
\end{align*}
\]

has no solutions (as you should verify).