Chapter 4
Stochastic Model for Thermal Fatigue Crack Growth

Abstract. In this chapter, the stochastic mathematical model is developed for the thermal fatigue crack growth phenomenon in the metallic pipe of a structural component. Any stochastic fatigue crack growth model used in time-reliability analysis must include a part means for incorporating randomness in service loads, and also another one, which should include a description of statistical characteristics of crack growth under constant amplitude loadings. Time-dependent fluctuation of temperature should be correlated with time dependent fluctuation of crack growth from the deterministic crack growth law. The number of loading cycles is a discrete variable with respect to time. When time-dependent stochastic analysis is conducted, the number of loading cycles is modified into a continuous variable by introducing an average cyclic rate. By stochastic analysis of a stationary Gaussian narrow-band process, we deal with the expected value of crack growth rate and expected rate of peak crossing (or mean rate of maxima) as well, in order to assess the thermal fatigue crack growth. The uncertainties in initial crack depth and Paris's law constants will be accounted by Monte Carlo simulation based on a properly limit state function (damage criterion). A method of crack growth assessment of linear elastic fracture mechanics (LEFM) for a stationary Gaussian narrow-band temperature fluctuation is given in this book. The model of stochastic fatigue crack growth is developed for cylindrical geometry, for which analytical solutions for temperature and associated elastic stresses were obtained in previous work. For the stochastic approach of crack growth due to random thermal fluctuations, only temporal incoherence is accounted and not any degree of spatial coherence has been taken into account.

4.1 The Main Steps of the Modeling

The parameters that affect structural fatigue performance include applied stress, geometry of structural details, properties of the material, and operating environment. A widely accepted empirical crack growth law originally was suggested by Paris, Gomez and Abderson (1961) [1]
where \( \frac{da}{dN} \) is the increment of fatigue crack advance per cycle, and \( \Delta K \) is the range of the stress intensity factor, which is related in linear elastic fracture mechanics to far-field nominal stress range, and component geometry factor. \( C \) and \( n \) are empirical constants dependent on material property and the environment. The regression analysis leading to Equation (4.1) only describes the crack growth rate in the median sense. Investigation of the randomness of fatigue crack growth rate under service load conditions must consider the statistical characteristics of crack growth law under constant amplitude loadings and the randomness of loadings that gives rise to fatigue under variable amplitude loads.

Any stochastic fatigue crack growth model used in time-reliability analysis must include a part means for incorporating randomness in service loads, and also another one, which should include a description of statistical characteristics of crack growth under constant amplitude loadings.

The selection of an appropriate stochastic model for fatigue crack growth depends on the nature of uncertainty to be interpreted. The application of stochastic fatigue analysis in this study is focused on the thermal fatigue damaging phenomenon in mixing tees of nuclear components. Time–dependent fluctuation of temperature should be correlated with time dependent fluctuation of crack growth from the deterministic crack growth law. The number of loading cycles is a discrete variable with respect to time. When time-dependent stochastic analysis is conducted, the number of loading cycles is modified into a continuous variable by introducing an average cyclic rate. In this case, we have [2]

\[
\frac{dA}{dt} = \frac{dA}{dN} \frac{dN}{dt} = \nu_p \frac{dA}{dN},
\]

(4.2)

in which \( A \) is the random flaw size (uppercase case denote random variables or processes) and \( \nu_p \) is the mean rate of maxima, that is constant for a stationary random process.

By means of stochastic analysis of a stationary Gaussian narrow-band process, we deal with the expected value of crack growth rate and expected rate of peak crossing (or mean rate of maxima) as well, in order to apply Equation (4.2) for thermal fatigue crack growth. The uncertainties in initial crack depth and Paris law constants will be accounted by Monte Carlo simulation based on a properly limit state function (damage criterion).

A method of crack growth assessment of linear elastic fracture mechanics (LEFM) for a stationary Gaussian narrow-band temperature fluctuation is given in this book. The model of stochastic fatigue crack growth is developed for cylindrical geometry, for which analytical solutions for temperature and associated elastic stresses have been obtained in previous works [7, 8, 9, 10, 11]. For the stochastic approach of crack growth due to random thermal fluctuations, only