Local, Polynomial $G^1$ PN Quads

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Abstract. In this paper, we introduce the concept of local, polynomial $G^1$ PN quads. These are degree bi-5 polynomial surface patches in Bernstein-Bézier form. As the classic PN patch, ours interpolates the vertices of a quadrilateral control polygon and is orthogonal to a normal specified at each vertex. In contrast to the original concept, the proposed quad is orthogonal to four (continuous) normal fields — one defined at each boundary. Each of these normal fields and the corresponding patch boundary are uniquely determined by the data at two adjacent vertices of the control polygon. Thus, the patch construction is local in the sense that it is based solely on the information provided at the four control vertices. In this way, it is easy to stitch together multiple quads to construct a manifold $G^1$ continuous surface of arbitrary topological type. In contrast to other approaches, vertices at which 3 or more than 4 patches meet do not require special treatment.

1 Introduction

If one needs a technique which makes a mesh look smooth, then PN patches [1] certainly are a reasonable choice. However, if a mesh has to be turned into a surface which has to be smooth, PN patches are not appropriate. Instead, other techniques could be used like subdivision surfaces [2], modified Coons and Gregory patches [3,4,5] or smooth spline macro-patch complexes [6].

Nevertheless, a PN patch has several nice characteristics. Being a bicubic Bézier spline it has a simple mathematical representation and can be evaluated efficiently. It is completely defined by a single polygon (a triangle or quad) plus normals specified at the vertices. Two PN patches whose defining polygons share an edge automatically share a (curved) boundary. Thus, it is easy for PN patches to join continuously. Moreover, the involved computations are local since once the control mesh and the normals are defined, the construction of each PN patch does not take its neighbors into account. To give the visual impression of smoothness, a separate quadratic normal field per patch is constructed and used for shading. They approximate the true normal field and when stitched together yield a continuous normal field over the entire mesh. This certainly improves the appearance, however, the shape of the resulting surface does not exhibit $G^1$ continuity across patch borders.

To eliminate this lack of smoothness, we introduce the $G^1$ PN quad. Although significantly slower in construction (solve a linear system) and not as fast in evaluation (degree bi-5 polynomial) as the classic PN quad, it has the following desirable properties.
Fig. 1. For the input (a), four boundaries and four corresponding normal fields are constructed as shown in (b). Based on that, the patch interior (c) is computed such that it is orthogonal to the normal fields at the boundary curves. Thus, the proposed quads can easily be stitched together to build a $G^1$ continuous surface as shown in (d).

- It is a single biquintic polynomial patch in Bernstein-Bézier form.
- It interpolates the vertices of a quadrilateral control polygon and is orthogonal to a normal prescribed at each vertex. Thus, it is defined by four points and normals.
- Two $G^1$ PN quads whose control polygons share an edge automatically share a (curved) border and have identical tangent planes along that border.
- Multiple patches can easily be stitched together to a $G^1$ continuous surface.
- Only local computations are involved in the sense that each patch is constructed without knowledge of its neighbors.
- The procedure does not depend on how many patches meet at a vertex.
- Manifold surfaces of arbitrary topological type can be built.
- The parametrization has no singularities.

What is particularly new about our approach is how the control points of a patch border are decoupled from the ones of the other borders. It has to do with the way how automatic $G^1$ continuity across patches is achieved. Each patch is orthogonal to the four user-defined normals at its corners and, moreover, to four continuous normal fields — one defined at each boundary. These orthogonality constraints lead to compatibility conditions which couple the control points of the borders and normal fields meeting at a vertex [7]. However, we construct each patch border and its corresponding normal field such that they automatically fulfill the compatibility conditions independent of adjacent boundary and normal curves. Furthermore, we achieve this without introducing singularities, resorting to rational functions or using multiple patches per quad. The price to pay is zero curvature at the patch corners which may lead to more or less visible flat spots.

In this paper, we are concerned with the geometric aspects of the proposed quads. Although interesting and relevant, we do not discuss GPU implementations, graphics pipeline concepts or other hardware related issues.

The rest of the paper is organized as follows. After reviewing previous work in Section 2, we describe the concept of $G^1$ PN quads in Section 3. Section 4 presents experimental results on smooth interpolation of quad meshes using the proposed quads. Conclusions are drawn in the final Section 5 of the paper.