Chapter 5
Waves and Vibrations

5.1 General Concepts

5.1.1 Types of waves

Our daily experience deals with sound waves, electromagnetic waves (as radio or light waves), deep or surface water waves, elastic waves in solid materials. Oscillatory phenomena manifest themselves also in less macroscopic and known contexts and ways. This is the case, for instance, of rarefaction and shock waves in traffic dynamics or of electrochemical waves in human nervous system and in the regulation of the heart beat. In quantum physics, everything can be described in terms of wave functions, at a sufficiently small scale.

Although the above phenomena share many similarities, they show several differences as well. For example, progressive water waves propagate a disturbance, while standing waves do not. Sound waves need a supporting medium, while electromagnetic waves do not. Electrochemical waves interact with the supporting medium, in general modifying it, while water waves do not.

Thus, it seems too hard to give a general definition of wave, capable of covering all the above cases, so that we limit ourselves to introduce some terminology and general concepts, related to specific types of waves. We start with one-dimensional waves.

a. Progressive or travelling waves are disturbances described by a function of the following form:

\[ u(x, t) = g(x - ct). \]

For \( t = 0 \), we have \( u(x, 0) = g(x) \), which is the “initial” profile of the perturbation. This profile propagates without change of shape with speed \(|c|\), in the positive (negative) \( x \)-direction if \( c > 0 \) \( (c < 0) \). We have already met this kind of waves in Chaps. 2 and 4.

b. Harmonic waves are particular progressive waves of the form

\[ u(x, t) = A \exp \{i(kx - \omega t)\}, \quad A, k, \omega \in \mathbb{R}. \quad (5.1) \]
It is understood that only the real part (or the imaginary part)

\[ A \cos (kx - \omega t) \]

is of interest, but the complex notation may often simplify the computations. In (5.1) we distinguish, considering for simplicity \( \omega \) and \( k \) positive:

- The wave amplitude \( |A| \).
- The wave number \( k \), which is the number of complete oscillations in the space interval \([0, 2\pi]\), and the wavelength

\[ \lambda = \frac{2\pi}{k}, \]

which is the distance between successive maxima (crest) or minima (troughs) of the waveform.
- The angular frequency \( \omega \), and the frequency

\[ f = \frac{\omega}{2\pi} \]

which is the number of complete oscillations in one second (Hertz) at a fixed space position.
- The wave or phase speed

\[ c_p = \frac{\omega}{k}, \]

which is the crests (or troughs) speed.

c. Standing waves are of the form

\[ u(x, t) = B \cos kx \cos \omega t. \]

In these disturbances, the basic sinusoidal wave, \( \cos kx \), is modulated by the time dependent oscillation \( B \cos \omega t \). A standing wave may be generated, for instance, by superposing two harmonic waves with the same amplitude, propagating in opposite directions:

\[ A \cos (kx - \omega t) + A \cos (kx + \omega t) = 2A \cos kx \cos \omega t. \] (5.2)

Consider now waves in dimension \( n > 1 \).