11. Ramanujan, modular equations, and approximations to pi or how to compute one billion digits of pi (1989)


Synopsis:
Here the authors reprise the history of \( \pi \), describe the recently discovered quadratically convergent algorithms, describe (briefly) the computational techniques required to compute, say, one billion digits of \( \pi \), and then explore in considerable detail the connections between these mathematical developments and the recently uncovered writings of Ramanujan, the hundredth anniversary of whose birth was celebrated in 1987. As the article observes, if Ramanujan had accomplished so much with paper, pencil and slateboard, how much further could he have seen if he had access to a modern symbolic computing environment such as Maple or Mathematica?

Interestingly, although not as asymptotically fast as the quadratically convergent formulas, Ramanujan discovered some very interesting formulas, e.g.

\[
\frac{1}{\pi} = \frac{\sqrt{8}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}},
\]

a variation of which was used by David and Gregory Chudnovsky to compute up to two billion digits (when accompanied with some very clever reorganization of the computation) in the early 1990s. Each term of this formula adds roughly eight correct digits.

Keywords: Algorithms, Computation, Elliptic Integrals, History, Modular Equations