4. The arithmetic-geometric mean and fast computation of elementary functions (1984)

Paper 4: J. M. Borwein and P. B. Borwein, "The arithmetic-geometric mean and fast computation of elementary functions," *SIAM Review*, vol. 26 (1984), p. 351–366. Copyright ©1984 Society for Industrial and Applied Mathematics. Reprinted with permission. All rights reserved.

Synopsis:

In this paper, brothers Jonathan and Peter Borwein present a review of the recently discovered (as of 1984) quadratically convergent formulas for π and elementary functions (including some new formulas of their own), complete with a rigorous derivation of all the requisite mathematics. The paper is thus an excellent self-contained tutorial on the theory of the arithmetic-geometric mean, quadratically convergent algorithms (including Newton's algorithm for computing square roots and roots of polynomials), and how these concepts lead to fast algorithms for π and elementary functions. They do this without needing to venture into incomplete elliptic integrals and Landen transforms, which were used to various degrees by earlier writers.

This paper has particular significance for the editors of this volume, as its appearance in *SIAM Review*, a publication mostly read in the applied mathematics and high-performance computing communities, led to Bailey becoming interested in these topics and joining with Jonathan and Peter Borwein in a multi-decade collaboration.

Keywords: Algorithms, Approximations, Arithmetic-Geometric Mean, Computation, History

© Springer International Publishing Switzerland 2016
D.H. Bailey, J.M. Borwein, *Pi: The Next Generation*,
DOI 10.1007/978-3-319-32377-0_4