7. The computation of π to 29,360,000 decimal digits using Borweins' quartically convergent algorithm (1988)

Paper 7: David H. Bailey, "The computation of pi to 29,360,000 decimal digits using Borweins' quartically convergent algorithm," *Mathematics of Computation*, vol. 50 (1988), p. 283–296. Reprinted by permission of the American Mathematical Society.

Synopsis:

This paper, written by one of the present editors, describes the computation of π using a set of formulas that at the time (1988) had just been discovered by Jonathan and Peter Borwein: Let $a_0 = 6 - 4\sqrt{2}$ and $y_0 = \sqrt{2} - 1$. Iterate

$$y_{k+1} = \frac{1 - (1 - y_k^4)^{1/4}}{1 + (1 - y_k^4)^{1/4}}$$

$$a_{k+1} = a_k(1 + y_{k+1})^4 - 2^{2k+3} y_{k+1}(1 + y_{k+1} + y_{k+1}^2).$$

Then a_k converges *quartically* to $1/\pi$: each successive iteration approximately quadruples the number of correct digits in the result.

Bailey also described in detail the computational techniques required to do such a long computation, such as the observation that a fast Fourier transform (FFT) can be employed to perform high-precision multiplication, and also presented the results of detailed statistical analyses of the digits.

Interestingly, this computation, which was performed on one of the original Cray-2 supercomputers while in a test mode, disclosed at least one bug in the hardware, which was subsequently rectified. In the wake of this finding, Cray employed a similar calculation as a test code to be run on new computers to ensure hardware integrity.

Keywords: Computation, Normality

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