Constrained Triangulation of 2D Shapes

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Abstract Algorithms for triangulating two dimensional shapes have been used as sub-problems in many application areas that include finite element analysis, geographic information systems, and geometric compression. We consider a constrained version of triangulation problem in which the objective is to increase the proportion of even degree vertices. We present an effective approach for generating triangulated polygons with increased number of even degree vertices. The proposed approach is based on the convex decomposition of polygon followed by 'diagonal flipping' operation.

Keywords Constrained triangulation · Mesh generation · Polygon decomposition

1 Introduction

Triangulation is the problem partitioning a two dimensional domain into triangles. The input is usually in the form of polygons and/or point sites and the output is a triangulated mesh. Triangle meshes have been extensively used in finite element analysis for obtaining approximate solution for fluid flow problem [6]. The quality of the solution depends on the quality of the generated mesh. A triangulated mesh with large proportion of non-skinny triangles is highly desired. In recent years it has been found that a triangulated polygon with increased number of even-degree vertices can be applied to tackle illumination problems [15].

In this paper we consider the problem of triangulating polygons with increased number of even degree vertices. We present how two well-known techniques of computational geometry can be used to obtain a triangulated polygon with high proportion of even-degree vertices. Specifically, we combine 'convex decomposition' and 'diagonal flipping' techniques to generate a triangulated mesh with high number of even-degree vertices. In Section 2, we present preliminaries and a brief

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review of recent progress made in understanding degree constrained triangulation problems. In Section 3, we present two approaches for triangulating simple polygons which can lead to increased number of even degree vertices. Finally, in Section 4, we discuss extensions of the proposed algorithms and direction for further investigation.

2 Preliminaries

A simple polygon is triangulated if its interior is partitioned into triangles by adding diagonals which do not interest on their interior. The problem of triangulating simple polygon is a well investigated problem and several algorithms for its solution have been reported [2,3,4], [7], [8], [13]. A linear time algorithm for triangulating a simple polygon is found in [3] which is rather complex for implementation. For practical implementation, randomized triangulation algorithms are preferred [7]. In recent years triangulation that satisfy additional properties have been considered [15]. One such property is the vertex degree requirement as defined below.

Definition 1: A simple polygon P is said to admit *even-degree triangulation* if every vertex in a triangulated graph of P is of even degree. Fig. 1 shows an example of even degree triangulation. The notion of odd-edge triangulation can be defined similarly.

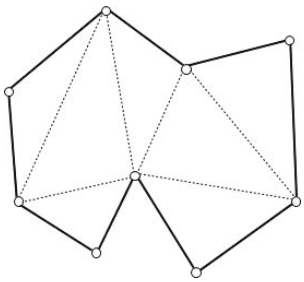

Fig. 1 Illustrating even-degree triangulation

An interesting issue in this regard is the possibility of triangulating a given polygon to make all vertices of even degree. This is stated in the following lemma.

Lemma 1: Not every polygon admits even degree triangulation.