DEEP BED FILTRATION THEORY
COMPARSED WITH EXPERIMENTS

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INTRODUCTION

Filtration is an important unit operation in gas cleaning and in the purification of molten metals.

From a fluid mechanical point of view removal of particles from a gas and from a melt are very similar. Therefore it is here attempted to present a theory valid for both gases and liquids (melts). "Cake-filtration" is not dealt with. In the presentation we start out by studying removal to a single sphere, this theory is then stretched to include a packed bed of spherical particles based on published experimental correlations. Finally it is indicated how the correlation can be applied to beds containing non-spherical particles - for instance ceramic open pore filters.

The theory is compared with some published experimental results concerning removal of particles from gases and melts. Also a comparison is made with metallographic studies of removal of inclusions to graphite sampling filters and Selen open pore ceramic filters.

THEORY

Collection efficiencies for single spheres.

The collection efficiency η, of a spherical particle to a single sphere (in an infinite fluid) can depend on the following variables: diameter of collector d , diameter of particle d , velocity of fluid relative to collector u (far away from the collector), velocity of particle relative to liquid u (for instance due to gravity or buoyancy effects), the diffusion coefficient of the particles D , the densities of particle and fluid ρ and ρ , respectively, and kinematic viscosity ν. Employing Buckingham II-theorem (1) it is found that the eight variables and three fundamental units m, s and k give us 8·3 = 5 independent dimensionless groups. We may for instance choose:

\[ Re = \frac{u_d}{\nu} \]
\[ \psi = \frac{N^2 \cdot Sc}{\rho_d \cdot \rho_f / 2} \]
\[ \psi_p = \frac{u_d}{\nu} \cdot \rho_f \]

Here Re gives the Reynolds number referred to the approach velocity u and collector diameter d .

For Re < 1 we are in the viscous flow region (2), while for Re > 1 a boundary layer is formed around the collector. N is a geometric group relating the size of the particle and collector. It plays an important role in describing the mechanism of removal by interception. Usually N << 1. This is the case especially in the removal of inclusions in melts where d is often in the range 0.1 to 10 μm while d usually lies between 100 to 1000 μm.

Thus the collection efficiency in the diffusion boundary layer δ, relative to the convective boundary layer thickness δ for (3):

\[ \delta = \frac{Sc}{1/3} \]

For particle sizes between 0.1 and 10 μm, Sc is very large both for gases and fluids. For instance for air and molten aluminium υ = 0.0110^-3 m/s and 0.410^-3 m/s respectively.

The Schmid number determines the thickness of the diffusion boundary layer δ to the convective boundary layer thickness δ

\[ D = \frac{3 \pi \cdot d \cdot \nu \cdot \nu_f}{k_f} \]

gives for air at room temperature and aluminium at 1000 K, respectively 3.6·10^-11 and 1.6·10^-12 m/s.

Thus the Sc-numbers for air and aluminium become Sc = 0.25·10^3 and 0.27·10^3. It is seen that the numerical values are not very different, around 3·10^3. This means that diffusion is controlled by the flow very close to the collector surface in both cases.

ψ called the inertial impaction parameter gives a ratio between inertial and viscous forces. In literature a number of theoretical calculations and experimental results (4) are found relating η and ψ. In the Calculations η is given as a function of ψ for various Re. Usually it is assumed that the particles have mass but no size, N = 0. For this inertial removal to be significant ψ > 0.03. In fluid filter systems this usually means that Re >> 1. In this case - assuming potential flow - the Langmuir-Blodgett calculations (5) may be employed.
For \( \psi < 0.7 \) we then have

\[
\eta_{lm} = (0.7 - 0.2) \cdot 0.4
\]  

(3)

It is seen that only when \( \psi > 0.04 \) do the particles cross the streamlines over to the collector surface.

Gravity \( \eta_g \)

\( \text{Re} \) is the Reynolds number referred to the particles. Usually \( \text{Re} \ll 1 \) so that Stokes’ law may be employed to calculate the velocity of the particle relative to the surrounding fluid. Stokes’ law gives for the hydrodynamic force \( F \) acting on a sphere in an infinite fluid

\[
F = 6\pi \eta \cdot u \cdot d_p
\]  

(4)

Eq (4) is not valid near a wall or near other particles. Additional forces that then must be taken into account tend to displace the particle away from the walls (6). It is difficult to determine how important these forces are. Here it is only assumed that eq (4) is valid. Equating eq (4) to gravity minus buoyancy forces then gives:

\[
u_p = \frac{(p_p - p_f) \cdot d_p^2 \cdot g}{18 \eta \cdot \nu}
\]  

(5)

where \( g \) is the acceleration of gravity, \( g = 9.81 \text{ m/s}^2 \).

For vertical flow through a filter one obtains

\[
\eta_v = \frac{u_p}{u_\infty}
\]  

(6)

Interception and diffusion, \( \eta_i, \eta_d \)

In the literature (7-10) the \( \eta \)'s for Brownian diffusion and interception are calculated for viscous flow, \( \text{Re} \ll 1 \) and for potential flow (\( \text{Re} \gg 1 \)). As pointed out previously, diffusion and interception effects are controlled by the flow very close to the collector — in the boundary layer for \( \text{Re} > 1 \). Therefore in this paper we first obtain relations for velocities in this boundary before calculating the \( \eta \) for the two mechanisms.

The stream function \( \eta \) is given by (11):

\[
\eta = \int_0^{2\pi} \int_0^{d_c} y \cdot d_y \cdot dy \cdot \bigg\{ 0 \cdot f_1'(v) - 0^3 \cdot f_3'(v)/3 + 0^5 \cdot f_5'(v) \cdot 3/120 - - - \bigg\}
\]  

(7)

where the \( f(v), f_3(v) \) and \( f_5(v) \) (11) are given as functions of

\[
v = \frac{\nu}{d_c \sqrt{6 \cdot \text{Re}_c}}
\]  

(8)

For \( y \) is the normal direction out from the spherical surface. \( \theta \) is the angle between the fluid velocity and \( y \). The tangential velocity is given by

\[
u = \frac{\partial v}{\partial y}
\]  

(9)

Insertion of eq (7) in eq (9) results in:

\[
u = \frac{3 \cdot u_\infty}{2} \bigg\{ 0 \cdot f_1'(v) - 0^3 \cdot f_3'(v)/3 + 0^5 \cdot f_5'(v) \cdot 3/120 - - - \bigg\}
\]  

(12)

When \( v = 0 \) we have the potential flow velocity distribution. The boundary conditions for this case (11) are \( f_1'(\infty) = 1, f_3'(\infty) = \frac{1}{3} \) and \( f_5'(\infty) = \frac{1}{3} \).

The collection efficiency for interception \( \eta_i \) may now be obtained from the relation

\[
\eta_i = \text{volume flow containing particles intercepted the sphere/volume flow approaching sphere}
\]  

(11)

The volume flow in the numerator is derived approximately as:

\[
\int_0^r u \cdot 2\pi \cdot r \cdot dy = 2\pi \cdot \nu \cdot r
\]  

(12)

where \( r \) is the distance from the symmetry axis (see fig. 1).