Chapter 12
On Electronically Restoring an Imperfect Vibratory Gyroscope to an Ideal State

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Dedicated to Charlotta E. Coetzee 1955 - 2012.

Abstract With regard to G.H. Bryan’s publication in 1890, we call the following Bryan’s law (or Bryan’s effect): "The vibration pattern of a revolving cylinder or bell revolves at a rate proportional to the inertial rotation rate of the cylinder or bell". Bryan’s factor is the proportionality constant that can be theoretically calculated for an ideal vibratory gyroscope (VG). If a perfectly symmetric VG is not ideal, that is, if imperfections and damping are present, then the precession rate (pattern rotation rate) depends on a number of factors. Indeed it depends on the rotation rate of the vehicle it is attached to, mass-stiffness and symmetry imperfections as well as any anisotropic damping (linear or nonlinear) that may be present in the VG. Assuming perfect axisymmetry for the VG, we show how to negate the effects of manufacturing mass-stiffness imperfections as well as the effects of any type of tangentially anisotropic damping that might occur. We achieve this by showing exactly how to symmetrically arrange an electronic array about the symmetry axis. This array consists of curved capacitors under a mixture of a constant (fixed) charge and a small meander charge. We show exactly how the fixed voltage on the capacitor should be adjusted in order to eliminate the frequency split caused by the mass-stiffness imperfection. Furthermore, we show how the meander voltages of the capacitors should be adjusted in order to maintain principal vibration, eliminate quadrature vibration and restore spurious pattern drift in the VG so that it obeys Bryan’s law, restoring the precession rate to the ideal rate so that Bryan’s factor can be used for calibration purpose. Equations of motion are derived in the form of averaged ODEs that provide us insight into VG behaviour.

Key words: Vibratory gyroscope · Capacitor array · Mass-stiffness imperfections · Anisotropic nonlinear damping · Nonlinear prestress
12.1 Introduction

Engineers exploit Bryan’s law (Bryan, 1890) for calibrating a navigational vibratory gyroscope (VG). However, when a VG is manufactured, inevitable manufacturing imperfections will be present if the manufacturing process is not stringent and/or the materials used are not chemically pure. Because a VG operates using Bryan’s effect and Bryan’s factor (the proportionality constant mentioned in the law), we will call the vibration dynamics of a VG an ideal state of vibration if Bryan’s effect is the same as that of a VG where manufacturing imperfections have been eliminated or controlled and no anisotropic damping is present or has been controlled.

It is folklore that a frequency split occurs affecting Bryan’s law in a VG if

- it is not perfectly axissymmetric about its axis of rotation (see e.g. Wang et al., 2015, 2016, for a list of citations) and/or
- mass imperfections are present (see e.g. Shatalov et al., 2011; Ma and Su, 2015, for a list of citations) and/or
- stiffness imperfections are present (see e.g. Joubert et al., 2011, 2015).

With the sophisticated computer-assisted turning machinery available nowadays we will assume that the VG is manufactured to a fine tolerance so that axissymmetry is established initially and we consequently eliminate this source of frequency splitting immediately from our considerations.

Isotropic damping does not affect Bryan’s law in a VG as discussed in Joubert et al. (2013), while the presence of any of the following three “anisotropies” in the VG does:

- mass imperfections,
- stiffness imperfections,
- any form of anisotropic damping.

The work presented here extends the work presented in Shatalov and Coetzee (2011) by

1. using a theory that embraces a not necessarily thin shelled VG;
2. introducing anisotropic nonlinear damping into the equations of motion;
3. extending the averaging process to include even order anisotropic damping;
4. including numerical experiments to visualise and explain VG behaviour and equations of motion;
5. clarifying symbol usage;
6. simplifying all equations;
7. showing exactly how the controlling electrodes should be manipulated; and
8. correcting a “concluding statement”.

A number of contemporary engineers are contemplating the manufacture of VGs that are not fused quartz hemispherical vibratory gyroscopes (HVGs). They use geometries that are easier to manufacture (such as bell, cylindrical, cone and paraboloid shapes) as well as materials such as metals that are readily machined, but that have a lower Q-factor than fused quartz. Recently, Ma and Su (2015), Liu et al.