In very generalized terms, a real function \( y = f(x) \) can be understood as a rule for assigning a real number \( y \) to a real number \( x \). Some functions play a central role in mathematics and their numerous applications, as well as in nature. Among others, these are the power functions of the form \( y = x^n \); the trigonometric functions \( y = \sin x, y = \tan x \), etc.; the exponential functions \( y = a^x \); and also their inverses.

Besides elementary functions, which are given with mathematical exactness, the so-called empirical functions also play an essential role in applied mathematics. They are mostly given by a number of measurement points (sampling points), and intermediate points are interpolated.

When solving problems of a physical, technical, or geometric nature, it is often necessary to employ the “derivative function” \( y' = f'(x) \) of the original function. The derivative is computed by means of simple rules of differential calculus, and it is indispensable in the solution of many problems.

In the age of computing, digital techniques are not only used to display functions, but also to differentiate them. This often leads to numerical problems, which will be discussed in this chapter. In general, the use of computers makes great sense, and it even allows us to solve so-called transcendental (non-algebraic) equations with sufficient precision.
6.1 Real functions and their inverses

In order to get acquainted with the problem, let us first discuss a diagram showing a series of measurements.

▸▸▸ Application: amount of lactate in blood related to physical work

The lactate value describes the relation between the concentration of lactic acid (lactate concentration) in blood and the intensity of physical work of a human body (Fig. 6.1).

![Lactate Profile Diagram](image)

The lactate concentration in the blood increases with increasing exertion. At a lactate value of up to 2 mmol per liter, the organism is working “aerobically”, receiving a large portion of the required energy through the oxygen-intensive burning of body fat – without depleting the valuable glycogen storage of the muscles and the liver too quickly (which can only be replenished over longer periods of time).

In the “anaerobic range”, the body increasingly needs to make use of these energy storages. Once the glycogen is fully depleted, the body becomes completely slack. Many long-distance runners experience this phenomenon if they do not hold back their tempo during the initial phase of their run, which causes them to reach the anaerobic phase too soon. On the axis of abscissas (usually the x axis), we notate the intensity of the physical work. On the axis of ordinates (usually the y axis), we notate a sufficient quantity of measured lactate values and connect the data points.

After this “warm-up example”, let us take a moment to clarify several terms so that we may approach the next considerations more exactly.

The real numbers \( \mathbb{R} \) are the set of all “decimal numbers”. Calculators and computers, for instance, usually work with finite decimal numbers – in other words, with a mere subset of the rational numbers and with functions that act on them.

Real numbers may be visualized on the real line, as the points on a straight line correspond exactly and unambiguously to real numbers. A segment corresponds to an interval of real numbers. The set \( \{ x \mid a \leq x \leq b \} \) is called a