6 Particle Swarm Optimization

6.1 The PSO Method

Inspired by animal behavior, Eberhart and Kennedy [49, 22] proposed in 1995 an optimization method called *Particle Swarm Optimization* (PSO). In this approach, a swarm of particles simultaneously explore a problem's search space with the goal of finding the global optimum configuration.

6.2 Principles of the Method

In PSO the *position* $\mathbf{x}_i$ of each particle i corresponds to a possible solution to the problem, with fitness $f(\mathbf{x}_i)$. In each iteration of the search algorithm the particles move as a function of their *velocity* $\mathbf{v}_i$. It is thus necessary that the structure of the search space allows such movement. For example, searching for the optimum of a continuous function in $\mathbb{R}^n$ offers such a possibility.

The particles' movement is similar to a flock of birds or a school of fish, or to a swarm of insects. In these examples, it is assumed that the animals move by following the individual in the group that knows the path to the optimum, perhaps a source of food. In addition, however, the individuals also follow their instinct and integrate the knowledge they have about the optimum into their movements.

In the PSO method two quantities $\mathbf{x}_i^{best}(t)$ and $\mathbf{B}(t)$ have to be defined and updated in each iteration. The first one, $\mathbf{x}_i^{best}(t)$, which is often called *particle-best*, corresponds to the best fitness point visited by particle i since the beginning of the search. The second quantity, $\mathbf{B}(t)$, called *global-best*, is the best fitness point reached by the population as a whole up to time step t:

$$\mathbf{B}(t) = \text{argmax}_{\mathbf{x}_i^{best}} f(\mathbf{x}_i^{best}(t))$$

In certain variants of PSO the *global-best* position $\mathbf{B}(t)$ is defined with respect to a sub-population to which a given individual belongs. The subgroup can be defined

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by a neighborhood relationship, either geographical or social. In this case, $\mathbf{B}$ will depend on i.

Therefore, as illustrated in Figure 6.1, the particles' movement in PSO is determined by three contributions. In the first place, there is a term accounting for the "inertia" of the particles: this term tends to keep them on their present trajectory. Second, they are attracted towards $\mathbf{B}(t)$, the global best. And third, they are also attracted towards their best fitness point $\mathbf{x}_i^{best}(t)$.

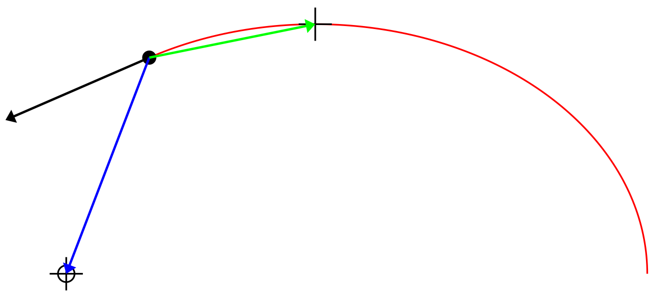

Fig. 6.1. The three forces acting on a PSO particle. In red, the particle's trajectory; in black, its present direction of movement; in blue, the attraction toward the *global-best*, and in green, the attraction towards the *particle-best*

Mathematically, the movement of a particle from one iteration to the next is described by the following formulas:

$$\begin{aligned}\mathbf{v}_i(t+1) &= \omega \mathbf{v}_i(t) + c_1 r_1(t+1)[\mathbf{x}_i^{best}(t) - \mathbf{x}_i(t)] \\ &\quad + c_2 r_2(t+1)[\mathbf{B}(t) - \mathbf{x}_i(t)] \\ \mathbf{x}_i(t+1) &= \mathbf{x}_i(t) + \mathbf{v}_i(t+1)\end{aligned} \tag{6.1}$$

where ω, c_1 and c_2 are constants to be specified, and r_1 and r_2 are pseudo-random numbers uniformly distributed in the interval $[0, 1]$. We remark that a different random number is used for each velocity component.

The c_1 parameter is called the *cognitive coefficient* since it reflects the individual's own "perception," and c_2 is called the *social coefficient*, since it takes into account the group's behavior. For example, $c_1 \approx c_2 \approx 2$ can be chosen. The ω parameter is the *inertia* constant, whose value is in general chosen as being slightly less than one.

Besides formulas (6.1), one must also impose the constraints that each velocity component must not be allowed to become arbitrarily large in absolute value. To this end, a $\mathbf{v}_{max}$ cutoff is prescribed. In the same way, the positions $\mathbf{x}_i$ are constrained to lie in a finite domain having diameter x_{max}.