COMPARISON OF SOME OPTIMIZATION METHODS AND THEIR APPLICATION IN THE SHAPE OPTIMIZATION OF LIFTING AIRFOILS

O. Pironneau – A. Vossinis
INRIA – Rocquencourt
B.P. 105, 78153 Le Chesnay Cedex – FRANCE

SUMMARY

We propose the use of nonlinear GMRES in order to solve the optimality conditions in unconstrained optimization problems. We show academic examples and industrial applications of shape optimization in potential flows.

INTRODUCTION

Nonlinear GMRES is a robust algorithm for solving systems of equations fast. Therefore, its use in Shape Optimization in Fluid Mechanics is attractive. We suggest two ways of use: either we solve by GMRES the optimality conditions or we minimize the cost function projected on the Krylov subspace used by GMRES.

A comparison of the above mentioned ways of use with some other optimization algorithms (namely: Steepest Descent, Powell’s Method and Least Squares Method) shows that GMRES for solving the optimality conditions (grad-GMRES) works best. The comparison is done with the help of an academic example of shape optimization in incompressible potential flow. We show also a simple application of airfoil shape optimization in the same kind of flow.

Then, we use grad-GMRES in two applications more industrial and less academic:
- Pressure recovery inverse problem for an airfoil in subsonic potential flow. The gradient calculation of the cost function is done by the use of an adjoint state (“exact computation” of the gradient).
• Shape Optimization of airfoils in transonic potential flow using an “exact com­putation” of the gradient of the cost function. We wish either to achieve a certain pressure distribution on the airfoil or to reduce the shock induced drag.

COMPARISON OF OPTIMIZATION METHODS

An optimization problem

We compare some optimization algorithms in the following shape optimization problem

Find the nozzle which realizes a certain velocity distribution on its walls for an incompressible, two-dimensional, irrotational and steady flow.

The flow is modeled by the use of a stream function $\Psi$, therefore the state equation is the following

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\begin{align}
-\Delta \Psi &= 0 \text{ in } \Omega \\
\frac{\partial \Psi}{\partial n} |_{r_1, r_2} &= 0 \\
\Psi |_{r_2} &= 1 \\
\Psi |_{r_1} &= 0
\end{align}
$$

(1)

where we assume that the flow velocity is parallel to the nozzle axis and uniform at the entrance and the exit of the nozzle and the wall and axis of the nozzle are streamlines (see Figure 1). $P^1$ finite elements are used for solving the state equation. The subscript $h$ denotes the discretization of the geometry and of the solution.

Figure 1: Notation on the nozzle section

The cost function is equal to

$$E(\Gamma_2) = \int_{\Gamma_2} | \nabla \Psi - \vec{v} |^2 \, d\Gamma,$$

(2)

with $\vec{v} = (v_1, v_2)$ the desired velocity distribution on $\Gamma_2$ and $\vec{c} = (-v_2, v_1)$. 

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