ELECTRIC FIELD DYNAMICS AT A CHARGED POINT
STRONG COUPLING LIMIT

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Abstract
The dynamics of electric fields at a charge $Q_0$ imbedded in a one component plasma of charges $Q$ is considered for conditions of large $Q_0/Q$ and large plasma parameter $\Gamma$. The complete response of the charge $Q_0$ to the surrounding plasma is formulated in terms of path integrals over the probability density functional for the electric field histories. Two applications of this formulation illustrating problems of both transport and plasma spectroscopy are noted. A set of Langevin equations for the position and velocity of the charge $Q_0$, and the total electric field at the charge are studied in the large charge, strong coupling limit. The analysis suggests a Gaussian, non-Markovian process for these variables. The resulting joint distribution of fields at two times and the generating functional for fields at many times are given. Finally, it is proposed that the set of observables can be extended slightly to yield an equivalent Gaussian-Markov process.

INTRODUCTION
An ion of charge $Q_0$ and mass $m_0$ is placed in a one component plasma (OCP) of ions with charge and mass, $Q$ and $m$. The entire system is taken to be charge neutral (uniform background) and at equilibrium. The objective here is to develop a complete description of the effects of the OCP on the imbedded charge. This objective encompasses static and dynamic effects, linear and nonlinear response, radiative and transport properties. Clearly such a program is too ambitious for progress beyond the formal level in general. However, it is noted that significant simplifications occur for conditions of strong coupling and large $Q_0/Q$. These simplifications lead to a tractable stochastic model that meets the stated objective.

Most theoretical studies of tagged particle motion in a many body system are based on kinetic theory (e.g., time correlation function/Green's function methods). While such methods are highly developed for linear response and one- or two-particle properties, they are not easily implemented for nonlinear coupling effects that involve many-particle or many-time properties. For such problems a quite different point of view is considered that focuses directly on the dynamics of a few

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relevant many-particle properties, rather than a detailed confrontation of the complete underlying microscopic N+1 particle dynamics. In the present case that property is the total electric field of the OCP at the ion, which represents the entire coupling of the ion to its environment. The formulation described below is exact, but its value lies primarily in suggesting approximations appropriate for complex problems.

The center of mass degrees of freedom obey Newton's equations,

\[ \frac{\partial \vec{r}(t)}{\partial t} = \vec{v}(t), \quad \frac{\partial \vec{v}(t)}{\partial t} = (Q/m_o) \vec{E}(t) \]  

(1.1)

where \( \vec{r}(t) \) and \( \vec{v}(t) \) are the position and velocity of the ion, and the microscopic electric field is given by,

\[ \vec{E} = \sum_{i=1}^{N} \vec{E}_i - \vec{E}_b \]  

(1.2)

Here, \( \vec{E}_i \) is the Coulomb electric field at the ion due to the \( i^{th} \) particle, and \( \vec{E}_b \) is the field of the uniform neutralizing background charge. If there is internal electronic structure to this ion, the main coupling of the internal coordinates to the OCP is through a dipole interaction, \( \vec{E}(t) \cdot \vec{d} \). The electric field is therefore the only relevant plasma variable determining the effects of the plasma on the ion for both its radiative and transport properties. To solve Eqs. (1.1) it is necessary to know the field at all times over some time interval of interest, \( 0 \leq t \leq T \). Every allowed function, \( \vec{E}(t) \), specified over this interval will be referred to as a field "history". The approach taken here is first to select a possible field history for given initial conditions, calculate the property of interest, and assign a probability for the chosen history. A final summation over all such choices provides the average value for the property considered. In this approach, the central unknown is the probability density functional for the electric field histories. A formal definition is obtained in terms of the corresponding joint probability density for the field to have specified values at \( M \) successive times,

\[ P_M[\vec{E}; T] \equiv \langle \prod_{p=0}^{M} \delta(\vec{E}(t_p) - \vec{E}(t_p)) \rangle, \quad t_p = pT/M \]  

(1.3)

\[ P[\vec{E}; T] = \lim_{M \to \infty} P_M[\vec{E}; T] \]  

(1.4)

The brackets in (1.3) denote an equilibrium ensemble average, and in the following we choose \( t_{i+1} > t_i = 0 \). The average of a functional of the field history, \( F[\vec{E}; T] \), is then given by,

\[ \langle F \rangle = \lim_{M \to \infty} \langle F \rangle \equiv \int \mathcal{D}[\vec{E}] P[\vec{E}; T] F[\vec{E}; T] \]  

(1.5)