10 Business Cycles: Discrete Time

10.1 Introduction

Although we for reasons given in general would prefer to work with continuous time models it must be admitted that there are certain advantages in displaying the details of chaos for discrete time models. This is so because, before the tools of analysis, such as symbolic dynamics, can be applied to the continuous models we need to construct the return map on the Poincaré section for the orbit investigated. This, however, means that we first have to integrate the system over a complete cycle. The details of such an integration can easily become just much too complex.

It is easier to display the chaotic regime for a discrete mapping where the recurrence map exists from the outset and we thus do not need this first step of constructing it. In economics, in particular the modelling of business cycles, discrete time always was the main framework. In what follows the Samuelson-Hicks multiplier-accelerator model will be discussed, though changed to a nonlinear format.

The original model proposed by Paul Samuelson in 1939 could generate cyclical change, vaguely resemblant of real business cycles, and the model remained the basis for business cycle modelling for many years. When it was solved in closed form the weakness of the linear model became apparent. Due to the linearity, it could only produce exponentially explosive or damped amplitudes.

The reader may also find it absurd that the model had a zero equilibrium, and that income would oscillate around this value, becoming negative in the depression phases of the cycle. As a matter of fact the complete model, however, also includes various “autonomous” expenditures, such as government spending, and other expenditures not induced by the business cycle. Those
Expenditures result in a positive equilibrium income, a stationary particular solution, from which the homogeneous system presented produces the deviations. This superposition principle also works in all the nonlinear variations on the model dealt with in this Chapter.

10.2 Investments

The principle of acceleration claims that for technological reasons the stock of capital always is in a given proportion to production, i.e. to real income. Investments, being defined as the rate of change of capital, would therefore be in proportion to the rate of change of income. In discrete time the rate of change is just the difference between income in two successive periods. Though being crude, in the sense that a Leontief type of technology without substitution is implied, the idea has a certain appeal due to its straightforwardness.

Investments, being among the determinants for income, the principle of acceleration provides for a feedback appropriate for formulating an interesting dynamics.

The underlying linear investment function was, however, questioned on factual grounds, as it implied active destruction of capital to keep the exact proportionality of capital stock to income whenever income declined at a faster rate than capital did in the complete absence of replacements. Sir John Hicks in 1950 suggested to replace the linear investment function by a piecewise linear function with upper and lower bounds. Whenever income decreased faster than the natural rate of capital depreciation, realised when no machinery or buildings at all that wore out were replaced, disinvestment just stayed at the negative value of this natural depreciation.

Likewise, whenever income increased so fast that other factors of production, such as labour, land, or raw materials, became limiting, then there was no point in investing beyond a certain maximum amount.

As an alternative Richard Goodwin suggested that the upper and lower bounds could be approached asymptotically by a hyperbolic tangent type of investment curve. Both the piecewise linear Hicksian function, and the smooth Goodwin alternative are shown on top of Fig. 10.1.

Both types can be approximated by a linear-cubic Taylor series expansion, and this variant in particular presents possibilities for an interesting dynamics. This Taylor series function is, however, back-bending, which is a feature