Quasi-matrix Deontic Logic

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Abstract. We use non-Kripkean quasi-matrix semantics for the formalization of the systems $S_{3d}$, $S_{3dp}$ and $S_{3dq}$ of deontic logic. The system $S_{3d}$ is weaker than the standard logic SDL. The semantics for $S_{3dp}$ represents combination of quasi-matrix semantics and the semantics of truth value gluts, which allows $S_{3dp}$ to avoid deontic explosion $O A \land O \neg A \supset OB$. The system $S_{3dq}$ rejects both deontic explosion and the formula $O A \land O \neg A \supset OA \land \neg OA$, thus it allows to consider deontic dilemmas without classical contradictions.

The systems $S_{5d}$, $S_{5dp}$ and $S_{5dq}$ in which the two types of deontic operators are used, namely, strong and weak obligation (permission), can be built as an extension of the correspondent systems $S_{3d}$, $S_{3dp}$ and $S_{3dq}$.

1 Quasi-Matrix Semantics for Modal Logic

Since the classical paper of Georg Henrik von Wright [19] deontic logic is considered as a special branch of modal logic. The so called standard deontic logic (SDL) can be obtained from the normal modal logic KD (system name is taken from B.Chellas [3]) by replacing the usual operators of necessity and possibility - respectively, box and diamond - by the “normative” operators, correspondingly, $O$ (is read as it is obligatory that, or it ought to be that), and $P$ (it is permissible that).

The system SDL contains all tautologies of classical propositional logic and the following axioms:

(SDLA1) $O (p \supset q) \supset (Op \supset Oq)$;
(SDLA2) $Op \supset Pp$;
(SDLA3) $Pp \equiv \neg O \neg p$;

Inference rules:
(SDLR1) Modus ponens;
(SDLR2) $\frac{Pp}{Op}$.

Operator $F$ (it is forbidden that) can be defined as $F p \overset{df}{=} \neg Pp$.

Semantics of SDL is based on the well-known Kripke model $M = (S, \pi, R)$, where $S$ is a set of possible worlds, $\pi$ is a truth assignment function assigning truth to the primitive propositions per each world, and $R \subseteq S \times S$ is a relation relating with each world a set of alternative worlds. Given a Kripke-model $M$ and a world $s \in S$, the modal operators are defined as follows:
\[(M, s) \models Op \text{ iff } \forall t(R(s, t) \Rightarrow (M, t) \models p)\]
\[(M, s) \models Pp \text{ iff } \exists t(R(s, t) & (M, t) \models p)\]
\[(M, s) \models Fp \text{ iff } \forall t(R(s, t) \Rightarrow (M, t) \neg \models p)\]

J. Kears (12) and Yu. Ivlev (11) in different ways suggested an idea of modal semantics which is completely different from Kripke-style semantics in that it does not use the idea of possible worlds. Instead of the alternative worlds one deals with the alternative interpretation quasi-functions formed by the given interpretation function (11). This approach allows to give table definitions for modal operators; there sometime can be vagueness in evaluations of modal formulas- in that case the value of the given formula \( A \) is considered to be “undetermined”, “alternative”- for instance, the value \( p/q \) means “either \( p \), or \( q \)”. Instead of an interpretation function, interpretation quasi-function takes only one single value from the fraction; in case of the value \( p/q \) there are two alternative quasi-interpreations, one in which \( A \) takes the value \( p \), and the other in which \( A \) takes \( q \). The formula \( A \) is true in the interpretation if and only if it is true in each alternative interpretation caused by the given interpretation.

The advantage of using quasi-matrix approach is that on its basis it is possible to consider the wide range of modal systems, which seems to include as a subset all known Kripkean modal logics and contains even more “intermediate” systems, for example the ones weaker then K, T, B, S4, etc. (some of that four valued modal systems were suggested in [11]). One can expect that the application of quasi-matrix approach to deontic logic can promote consideration of some additional aspects of deontic matters. Another good point of quasi-matrix semantics is that it allows to define the properties of modal (deontic) logic under construction beforehand, just in the table definitions of modal operators. In particular, in the described below systems there is an opportunity to consider separately the properties of the action sentences (acts) and to enter special deontic connectives by means of which the complex acts are formed. The properties of deontic connectives are set by table definitions in accordance with the preliminary informal discussions about the character of action of that connectives.

The considered below deontic system \( S_{3d} \) represents modification of three-valued quasi-matrix deontic logic suggested in [11] and is weaker than the standard deontic logic SDL, because the axiom SDLA1 the rule SDLR2 are no longer valid in \( S_{3d} \). In the next part we bring the intuitions on the system \( S_{3d} \).

2 Intuitions

First of all, the distinction between terms and formulas of deontic system \( S_{3d} \) has the same motivation as it was given in logic for normative propositions [1] which has been constructed on the basis of possible-world semantics. The reason for such distinction is that “deontic operators require as operands descriptions of actions (action sentences) but once deontic operator is applied to an action sentence the resulting deontic sentence is no longer a description of an action but a normative qualification of the action described by the action sentence.