Mott reached closure in his exploration of geometric fragmentation statistics early in his third internal report. In the remainder of this report he undertook a seminal investigation of the fragmentation of exploding shells, and developed a statistical theory of dynamic fragmentation elegant in its formulation and insightful in the physics explored. His theoretical effort has been noted in numerous subsequent studies in dynamic fragmentation but has received little in-depth study. Consequently, the fragmentation theory of Mott now over 60 years in the literature has been neither validated nor refuted. Efforts in the present section attempt to assess and broaden the physical principles of dynamic fragmentation first proposed by Mott. The efforts go beyond the initial analysis of Mott, however, both in the range of fracture processes, as well as in the analytic development.

3.1 Statistical Theory of Mott

The dynamic fracture analysis pursued by Mott is decidedly one-dimensional. It is best visualized as that of a uniformly stretching rod or expanding ring such as illustrated in Fig. 3.1. The model can be usefully abstracted to fragmentation applications, such as a rapidly expanding cylinder in which the circumferential stretching rate substantially exceeds the axial, or a one-dimensional spall event within a body experiencing increasing tension within a region of uniform axial velocity gradient. Here, for clarity, the model exploration will focus on a stretching filament of material of unit cross section as depicted by the expanding ring in Fig. 3.1. Prior to fracture, the body is uniformly stretched to an axial strain $\varepsilon$ which is increasing at a constant strain rate $\dot{\varepsilon}$.

Mott considered the body to be rigid perfectly plastic and straining in tension under a constant flow stress $Y$. The Mott kinematic conditions will be referred to as plastic fracture. Tensile loading in which the body remains elastic up to the point of fracture will also be considered (elastic fracture). Here
Fig. 3.1. The one-dimensional Mott problem. A one-dimensional ring of material undergoes outward expansion at constant velocity, $u$. Prior to fracture response of the body is uniform tensile stretching at a strain rate $\dot{\varepsilon} = u/r$. Instantaneous fracture occurs at random sites and waves originate at points of fracture which propagate at finite speeds, relieving tensile stress and further stretching. Strain-dependent fracture continues only in regions not yet encompasses by the stress-relieved waves.

tensile stress is related to strain and strain rate according to $\sigma = E\varepsilon = E\dot{\varepsilon}t$ where $E$ is the appropriate elastic modulus.

At onset of breakup fractures are considered to occur at random in both time (or equivalently strain) and in spatial location on the stretching body as illustrated in Fig. 3.1. Following Mott it is assumed that fractures occur instantaneously relieving the tensile stress at the point of fracture to zero. Thus fracture resistance at the point of breakage and corresponding fracture energy during the breakage process is ignored.

Mott argued that the fracture energy was not significant. Rather, he proposed that the statistical nature of the fracture process determined both the characteristic fragment size, as well as the distribution in fragment sizes.

Mott’s assumption of both instantaneous fracture and the insignificance of fracture energy can, and should, be examined further. This issue will be investigated in some detail in a later section.

Mott used observations of fracture in notched-bar specimens of steels to support the theoretical approach. He noted that the reduction in the cross-sectional area (the strain) before fracture was not the same from test to test. Scatter in the strain to fracture of a few percent over a number of tests was observed. He then proposed that strain to fracture was a random variable in