Chapter 1

Set Theory. Relations. Functions

1.1 $x \in A, \quad x \notin B$

The element x belongs to the set A, but x does not belong to the set B.

1.2 $A \subset B \iff$ Each element of A is also an element of B.

A is a *subset* of B. Often written $A \subseteq B$.

1.3 If S is a set, then the set of all elements x in S with property $\varphi(x)$ is written

$$A = \{x \in S : \varphi(x)\}$$

If the set S is understood from the context, one often uses a simpler notation:

$$A = \{x : \varphi(x)\}$$

General notation for the specification of a set. For example, $\{x \in \mathbb{R} : -2 \leq x \leq 4\} = [-2, 4]$.

1.4 The following logical operators are often used when P and Q are statements:

- $P \wedge Q$ means "P and Q"
- $P \vee Q$ means "P or Q"
- $P \Rightarrow Q$ means "if P then Q" (or "P only if Q", or "P implies Q")
- $P \Leftarrow Q$ means "if Q then P"
- $P \Leftrightarrow Q$ means "P if and only if Q"
- $\neg P$ means "not P"

Logical operators. (Note that "P or Q" means "either P or Q or both".)

1.5

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Truth table for logical operators. Here T means "true" and F means "false".

1.6

- P is a *sufficient condition* for Q: $P \Rightarrow Q$
- Q is a *necessary condition* for P: $P \Rightarrow Q$
- P is a *necessary and sufficient condition* for Q: $P \Leftrightarrow Q$

Frequently used terminology.

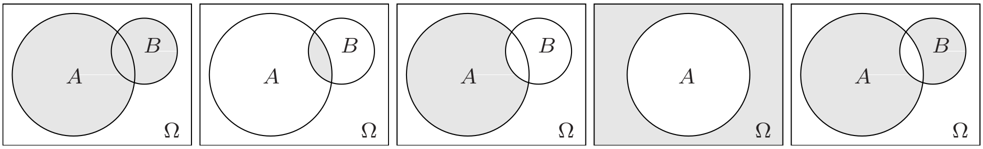
1.7

$$
\begin{aligned}
A \cup B &= \{x : x \in A \lor x \in B\} \quad (A \textit{ union } B) \\
A \cap B &= \{x : x \in A \land x \in B\} \quad (A \textit{ intersection } B) \\
A \setminus B &= \{x : x \in A \land x \notin B\} \quad (A \textit{ minus } B) \\
A \triangle B &= (A \setminus B) \cup (B \setminus A) \quad (\textit{symmetric difference})
\end{aligned}
$$

If all the sets in question are contained in some "universal" set Ω, one often writes $\Omega \setminus A$ as

$$A^c = \{x : x \notin A\} \quad (\text{the } \textit{complement} \text{ of } A)$$

Basic set operations. $A \setminus B$ is called the *difference* between A and B. An alternative symbol for A^c is $\complement A$.



$A \cup B$ $\quad$ $A \cap B$ $\quad$ $A \setminus B$ $\quad$ A^c $\quad$ $A \triangle B$

1.8

$$
\begin{aligned}
A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \\
A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\
A \triangle B &= (A \cup B) \setminus (A \cap B) \\
(A \triangle B) \triangle C &= A \triangle (B \triangle C) \\
A \setminus (B \cup C) &= (A \setminus B) \cap (A \setminus C) \\
A \setminus (B \cap C) &= (A \setminus B) \cup (A \setminus C) \\
A \cup B)^c &= A^c \cap B^c \\
(A \cap B)^c &= A^c \cup B^c
\end{aligned}
$$

Important identities in set theory. The last four identities are called *De Morgan's laws.*

1.9

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) : a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}$$

The *Cartesian product* of the sets A_1, A_2, …, A_n.

1.10

$$R \subset A \times B$$

Any subset R of $A \times B$ is called a *relation* from the set A into the set B.

1.11

$$
\begin{aligned}
xRy &\iff (x, y) \in R \\
x \not R y &\iff (x, y) \notin R
\end{aligned}
$$

Alternative notations for a relation and its negation. We say that x is in R-relation to y if $(x, y) \in R$.

1.12

- $\operatorname{dom}(R) = \{a \in A : (a, b) \in R \text{ for some } b \text{ in } B\} = \{a \in A : aRb \text{ for some } b \text{ in } B\}$
- $\operatorname{range}(R) = \{b \in B : (a, b) \in R \text{ for some } a \text{ in } A\} = \{b \in B : aRb \text{ for some } a \text{ in } A\}$

The *domain* and *range* of a relation.