# Chapter 25

# Properties of cost and profit functions

25.1 $$C(\mathbf{w}, y) = \min_{\mathbf{x}} \sum_{i=1}^{n} w_i x_i \quad \text{when} \quad f(\mathbf{x}) = y$$

*Cost minimization.* One output. $f$ is the production function, $\mathbf{w} = (w_1, \dots, w_n)$ are factor prices, $y$ is output and $\mathbf{x} = (x_1, \dots, x_n)$ are factor inputs. $C(\mathbf{w}, y)$ is the cost function.

25.2 $$C(\mathbf{w}, y) = \begin{cases} \text{The minimum cost of producing} \\ y \text{ units of a commodity when fac-} \\ \text{tor prices are } \mathbf{w} = (w_1, \dots, w_n). \end{cases}$$

The *cost function.*

25.3
- $C(\mathbf{w}, y)$ is increasing in each $w_i$.
- $C(\mathbf{w}, y)$ is homogeneous of degree 1 in $\mathbf{w}$.
- $C(\mathbf{w}, y)$ is concave in $\mathbf{w}$.
- $C(\mathbf{w}, y)$ is continuous in $\mathbf{w}$ for $\mathbf{w} > \mathbf{0}$.

Properties of the cost function.

25.4 $$x_i^*(\mathbf{w}, y) = \begin{cases} \text{The cost minimizing choice of} \\ \text{the } i\text{th input factor as a func-} \\ \text{tion of the factor prices } \mathbf{w} \text{ and} \\ \text{the production level } y. \end{cases}$$

*Conditional factor demand functions.* $\mathbf{x}^*(\mathbf{w}, y)$ is the vector $\mathbf{x}^*$ that solves the problem in (25.1).

25.5
- $x_i^*(\mathbf{w}, y)$ is decreasing in $w_i$.
- $x_i^*(\mathbf{w}, y)$ is homogeneous of degree 0 in $\mathbf{w}$.

Properties of the conditional factor demand function.

25.6 $$\frac{\partial C(\mathbf{w}, y)}{\partial w_i} = x_i^*(\mathbf{w}, y), \quad i = 1, \dots, n$$

*Shephard's lemma.*

25.7 $$\left( \frac{\partial^2 C(\mathbf{w}, y)}{\partial w_i \partial w_j} \right)_{(n \times n)} = \left( \frac{\partial x_i^*(\mathbf{w}, y)}{\partial w_j} \right)_{(n \times n)}$$
is symmetric and negative semidefinite.

Properties of the *substitution matrix.*

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| 25.8 | $\pi(p, \mathbf{w}) = \max_{\mathbf{x}} \left( pf(\mathbf{x}) - \sum_{i=1}^{n} w_i x_i \right)$ | The profit maximizing problem of the firm. $p$ is the price of output. $\pi(p, \mathbf{w})$ is the *profit function*. |
| 25.9 | $\pi(p, \mathbf{w}) = \begin{cases} \text{The maximum profit as a function} \\ \text{of the factor prices } \mathbf{w} \text{ and the out-} \\ \text{put price } p. \end{cases}$ | The profit function. |
| 25.10 | $\pi(p, \mathbf{w}) \equiv \max_{y} \left( py - C(\mathbf{w}, y) \right)$ | The profit function in terms of costs and revenue. |
| 25.11 | • $\pi(p, \mathbf{w})$ is increasing in $p$.<br>• $\pi(p, \mathbf{w})$ is homogeneous of degree 1 in $(p, \mathbf{w})$.<br>• $\pi(p, \mathbf{w})$ is convex in $(p, \mathbf{w})$.<br>• $\pi(p, \mathbf{w})$ is continuous in $(p, \mathbf{w})$ for $\mathbf{w} > \mathbf{0}$, $p > 0$. | Properties of the profit function. |
| 25.12 | $x_i(p, \mathbf{w}) = \begin{cases} \text{The profit maximizing choice of} \\ \text{the } i\text{th input factor as a function} \\ \text{of the price of output } p \text{ and the} \\ \text{factor prices } \mathbf{w}. \end{cases}$ | The *factor demand functions*. $\mathbf{x}(p, \mathbf{w})$ is the vector $\mathbf{x}$ that solves the problem in (25.8). |
| 25.13 | • $x_i(p, \mathbf{w})$ is decreasing in $w_i$.<br>• $x_i(p, \mathbf{w})$ is homogeneous of degree 0 in $(p, \mathbf{w})$.<br>The *cross-price effects* are symmetric:<br>$\dfrac{\partial x_i(p, \mathbf{w})}{\partial w_j} = \dfrac{\partial x_j(p, \mathbf{w})}{\partial w_i}, \quad i, j = 1, \ldots, n$ | Properties of the factor demand functions. |
| 25.14 | $y(p, \mathbf{w}) = \begin{cases} \text{The profit maximizing output as} \\ \text{a function of the price of output } p \\ \text{and the factor prices } \mathbf{w}. \end{cases}$ | The *supply function* $y(p, \mathbf{w}) = f(\mathbf{x}(p, \mathbf{w}))$ is the $y$ that solves the problem in (25.10). |
| 25.15 | • $y(p, \mathbf{w})$ is increasing in $p$.<br>• $y(p, \mathbf{w})$ is homogeneous of degree 0 in $(p, \mathbf{w})$. | Properties of the supply function. |
| 25.16 | $\dfrac{\partial \pi(p, \mathbf{w})}{\partial p} = y(p, \mathbf{w})$<br>$\dfrac{\partial \pi(p, \mathbf{w})}{\partial w_i} = -x_i(p, \mathbf{w}), \quad i = 1, \ldots, n$ | *Hotelling's lemma.* |