Chapter 26

Consumer theory

26.1 A *preference relation* $\succeq$ on a set X of commodity vectors $\mathbf{x} = (x_1, \ldots, x_n)$ is a complete, reflexive, and transitive binary relation on X with the interpretation

$$\mathbf{x} \succeq \mathbf{y} \quad \text{means:} \quad \mathbf{x} \text{ is at least as good as } \mathbf{y}$$

Definition of a preference relation. For binary relations, see (1.16).

Relations derived from $\succeq$:

26.2
- $\mathbf{x} \sim \mathbf{y} \iff \mathbf{x} \succeq \mathbf{y}$ and $\mathbf{y} \succeq \mathbf{x}$
- $\mathbf{x} \succ \mathbf{y} \iff \mathbf{x} \succeq \mathbf{y}$ but not $\mathbf{y} \succeq \mathbf{x}$

$\mathbf{x} \sim \mathbf{y}$ is read "$\mathbf{x}$ is *indifferent* to $\mathbf{y}$", and $\mathbf{x} \succ \mathbf{y}$ is read "$\mathbf{x}$ is *(strictly) preferred* to $\mathbf{y}$".

26.3
- A function $u : X \to \mathbb{R}$ is a *utility function representing the preference relation* $\succeq$ if
$$\mathbf{x} \succeq \mathbf{y} \iff u(\mathbf{x}) \geq u(\mathbf{y})$$
- For any strictly increasing function $f : \mathbb{R} \to \mathbb{R}$, $u^*(\mathbf{x}) = f(u(\mathbf{x}))$ is a new utility function representing the same preferences as $u(\cdot)$.

A property of utility functions that is invariant under every strictly increasing transformation, is called *ordinal*. *Cardinal* properties are those *not* preserved under strictly increasing transformations.

26.4 Let $\succeq$ be a complete, reflexive, and transitive preference relation that is also *continuous* in the sense that the sets

$$\{\mathbf{x} : \mathbf{x} \succeq \mathbf{x}^0\} \text{ and } \{\mathbf{x} : \mathbf{x}^0 \succeq \mathbf{x}\}$$

are both closed for all $\mathbf{x}^0$ in X. Then $\succeq$ can be represented by a continuous utility function.

Existence of a continuous utility function. For properties of relations, see (1.16).

26.5 *Utility maximization* subject to a budget constraint:

$$\max_{\mathbf{x}} u(\mathbf{x}) \text{ subject to } \mathbf{p} \cdot \mathbf{x} = \sum_{i=1}^{n} p_i x_i = m$$

$\mathbf{x} = (x_1, \ldots, x_n)$ is a vector of (quantities of) commodities, $\mathbf{p} = (p_1, \ldots, p_n)$ is the price vector, m is income, and u is the utility function.

26.6	$v(\mathbf{p}, m) = \max_{\mathbf{x}}\{u(\mathbf{x}) : \mathbf{p}\cdot\mathbf{x} = m\}$	The *indirect utility function*, $v(\mathbf{p}, m)$, is the maximum utility as a function of the price vector $\mathbf{p}$ and the income m.
26.7	• $v(\mathbf{p}, m)$ is decreasing in $\mathbf{p}$. • $v(\mathbf{p}, m)$ is increasing in m. • $v(\mathbf{p}, m)$ is homogeneous of degree 0 in $(\mathbf{p}, m)$. • $v(\mathbf{p}, m)$ is quasi-convex in $\mathbf{p}$. • $v(\mathbf{p}, m)$ is continuous in $(\mathbf{p}, m)$, $\mathbf{p} > \mathbf{0}$, $m > 0$.	Properties of the indirect utility function.
26.8	$\omega = \dfrac{u_1'(\mathbf{x})}{p_1} = \cdots = \dfrac{u_n'(\mathbf{x})}{p_n}$	First-order conditions for problem (26.5), with ω as the associated Lagrange multiplier.
26.9	$\omega = \dfrac{\partial v(\mathbf{p}, m)}{\partial m}$	ω is called the *marginal utility* of money.
26.10	$x_i(\mathbf{p}, m) = \begin{cases} \text{the optimal choice of the } i\text{th commodity as a function of the price vector } \mathbf{p} \text{ and the income } m. \end{cases}$	The *consumer demand functions*, or *Marshallian demand functions*, derived from problem (26.5).
26.11	$\mathbf{x}(t\mathbf{p}, tm) = \mathbf{x}(\mathbf{p}, m)$, t is a positive scalar.	The demand functions are homogeneous of degree 0.
26.12	$x_i(\mathbf{p}, m) = -\dfrac{\dfrac{\partial v(\mathbf{p}, m)}{\partial p_i}}{\dfrac{\partial v(\mathbf{p}, m)}{\partial m}}, \quad i = 1, \dots, n$	*Roy's identity.*
26.13	$e(\mathbf{p}, u) = \min_{\mathbf{x}}\{\mathbf{p}\cdot\mathbf{x} : u(\mathbf{x}) \geq u\}$	The *expenditure function*, $e(\mathbf{p}, u)$, is the minimum expenditure at prices $\mathbf{p}$ for obtaining at least the utility level u.
26.14	• $e(\mathbf{p}, u)$ is increasing in $\mathbf{p}$. • $e(\mathbf{p}, u)$ is homogeneous of degree 1 in $\mathbf{p}$. • $e(\mathbf{p}, u)$ is concave in $\mathbf{p}$. • $e(\mathbf{p}, u)$ is continuous in $\mathbf{p}$ for $\mathbf{p} > \mathbf{0}$.	Properties of the expenditure function.