Chapter 28

Topics from finance and growth theory

28.1 $S_t = S_{t-1} + rS_{t-1} = (1+r)S_{t-1}, \quad t = 1, 2, \ldots$

In an account with interest rate r, an amount S_{t-1} increases after one period to S_t.

28.2 The *compound amount* S_t of a *principal* S_0 at the end of t periods at the interest rate r compounded at the end of each period is

$$S_t = S_0(1+r)^t$$

Compound interest. (The solution to the difference equation in (28.1).)

28.3 The amount S_0 that must be invested at the interest rate r compounded at the end of each period for t periods so that the compound amount will be S_t, is given by

$$S_0 = S_t(1+r)^{-t}$$

S_0 is called the *present value* of S_t.

28.4 When interest is compounded n times a year at regular intervals at the rate of r/n per period, then the effective annual interest is

$$\left(1 + \frac{r}{n}\right)^n - 1$$

Effective annual rate of interest.

28.5
$$A_t = \frac{R}{(1+r)^1} + \frac{R}{(1+r)^2} + \cdots + \frac{R}{(1+r)^t} = R\frac{1-(1+r)^{-t}}{r}$$

The *present value* A_t of an *annuity* of R per period for t periods at the interest rate of r per period. Payments at the end of each period.

28.6 The present value A of an annuity of R per period for an infinite number of periods at the interest rate of r per period, is

$$A = \frac{R}{(1+r)^1} + \frac{R}{(1+r)^2} + \cdots = \frac{R}{r}$$

The present value of an infinite annuity. First payment after one period.

28.7 $$T = \frac{\ln\left(\dfrac{R}{R - rA}\right)}{\ln(1+r)}$$

The number T of periods needed to pay off a loan of A with periodic payment R and interest rate r per period.

28.8 $$S_t = (1+r)S_{t-1} + (y_t - x_t), \quad t = 1, 2, \ldots$$

In an account with interest rate r, an amount S_{t-1} increases after one period to S_t, if y_t are the deposits and x_t are the withdrawals in period t.

28.9 $$S_t = (1+r)^t S_0 + \sum_{k=1}^{t} (1+r)^{t-k}(y_k - x_k)$$

The solution of equation (28.8)

28.10 $$S_t = (1+r_t)S_{t-1} + (y_t - x_t), \quad t = 1, 2, \ldots$$

Generalization of (28.8) to the case with a variable interest rate, r_t.

28.11 $$D_k = \frac{1}{\prod_{s=1}^{k}(1+r_s)}$$

The *discount factor* associated with (28.10). (Discounted from period k to period 0.)

28.12 $$R_k = \frac{D_k}{D_t} = \prod_{s=k+1}^{t}(1+r_s)$$

The *interest factor* associated with (28.10).

28.13 $$S_t = R_0 S_0 + \sum_{k=1}^{t} R_k(y_k - x_k)$$

The solution of (28.10). R_k is defined in (28.12). (Generalizes (28.9).)

28.14 $$a_0 + \frac{a_1}{1+r} + \frac{a_2}{(1+r)^2} + \cdots + \frac{a_n}{(1+r)^n} = 0$$

r is the *internal rate of return* of an investment project. Negative a_t represents outlays, positive a_t represents receipts at time t.

28.15 If $a_0 < 0$ and $a_1, \ldots, a_n$ are all ≥ 0, then (28.14) has a unique solution $1 + r^* > 0$, i.e. a unique internal rate of return $r^* > -1$. The internal rate of return is positive provided $\sum_{i=0}^{n} a_i > 0$.

Consequence of Descartes's rule of signs (2.12).

28.16 $$A_0 = a_0,\ A_1 = a_0 + a_1,\ A_2 = a_0 + a_1 + a_2,\ \ldots,$$
$$A_n = a_0 + a_1 + \cdots + a_n$$

The *accumulated cash flow* associated with (28.14).