Chapter 5

Elasticities. Elasticities of substitution

5.1 \( \text{El}_x f(x) = \frac{x}{f(x)} f'(x) = \frac{x}{y} \frac{dy}{dx} = \frac{d(\ln y)}{d(\ln x)} \)

5.2

\[ \begin{align*}
\text{Marshall’s rule: } & \text{To find the elasticity of } y = f(x) \text{ w.r.t. } x \text{ at the point } P \text{ in the figure, first}\n& \text{draw the tangent to the curve at } P. \text{ Measure}\n& \text{the distance } A_y \text{ from } P \text{ to the point where the}\n& \text{tangent intersects the } y\text{-axis, and the distance}\n& A_x \text{ from } P \text{ to where the tangent intersects the}\n& x\text{-axis. Then } \text{El}_x f(x) = \pm \frac{A_y}{A_x}.\n\end{align*} \]

5.3

- If \(|\text{El}_x f(x)| > 1\), then \( f \) is elastic at \( x \).
- If \(|\text{El}_x f(x)| = 1\), then \( f \) is unitary elastic at \( x \).
- If \(|\text{El}_x f(x)| < 1\), then \( f \) is inelastic at \( x \).
- If \(|\text{El}_x f(x)| = 0\), then \( f \) is completely inelastic at \( x \).

5.4 \( \text{El}_x (f(x)g(x)) = \text{El}_x f(x) + \text{El}_x g(x) \)

5.5 \( \text{El}_x \left( \frac{f(x)}{g(x)} \right) = \text{El}_x f(x) - \text{El}_x g(x) \)

El\( _x f(x) \), the \textit{elasticity} of \( y = f(x) \) w.r.t. \( x \), is approximately the percentage change in \( f(x) \) corresponding to a one per cent increase in \( x \).

Illustration of Marshall’s rule.

Marshall’s rule. The distances are measured positive. Choose the plus sign if the curve is increasing at \( P \), the minus sign in the opposite case.

Terminology used by many economists.

General rules for calculating elasticities.
5.7 \( \text{El}_f (f(x) \pm g(x)) = \frac{f(x) \text{El}_f f(x) \pm g(x) \text{El}_f g(x)}{f(x) \pm g(x)} \) General rules for calculating elasticities.

5.8 \( \text{El}_f g(x) = \text{El}_u f(u) \text{El}_x u, \quad u = g(x) \)

If \( y = f(x) \) has an inverse function \( x = g(y) = f^{-1}(y) \), then, with \( y_0 = f(x_0) \),

5.9 \( \text{El}_y x = \frac{y \frac{dx}{dy}}{x} \), i.e. \( \text{El}_y g(y_0)) = \frac{1}{\text{El}_f f(x_0)} \)

The elasticity of the inverse function.

5.10 \( \text{El}_x A = 0, \quad \text{El}_x x^a = a, \quad \text{El}_x e^x = x. \)

(\( A \) and \( a \) are constants, \( A \neq 0 \).) Special rules for elasticities.

5.11 \( \text{El}_x \sin x = \frac{x}{\cos x}, \quad \text{El}_x \cos x = -\frac{x}{\sin x} \cos x \)

5.12 \( \text{El}_x \tan x = \frac{x}{\sin x \cos x}, \quad \text{El}_x \cot x = -\frac{x}{\sin x} \sin x \cos x \)

5.13 \( \text{El}_x \ln x = \frac{1}{\ln x}, \quad \text{El}_x \log_a x = \frac{1}{\ln x} \)

The partial elasticity of \( f(x) = f(x_1, \ldots, x_n) \) w.r.t. \( x_i, i = 1, \ldots, n \).

5.14 \( \text{El}_i f(x) = \text{El}_{x_i} f(x) = \frac{x_i}{f(x)} \frac{\partial f(x)}{\partial x_i} \)

If \( z = F(x_1, \ldots, x_n) \) and \( x_i = f_i(t_1, \ldots, t_m) \) for \( i = 1, \ldots, n \), then for all \( j = 1, \ldots, m \),

5.15 \( \text{El}_{t_j} z = \sum_{i=1}^{n} \text{El}_i F(x_1, \ldots, x_n) \text{El}_{t_j} x_i \)

The chain rule for elasticities.

The directional elasticity of \( f \) at \( x \), in the direction of \( x/\|x\| \), is

5.16 \( \text{El}_a f(x) = \frac{\|x\|}{f(x)} f'_a(x) = \frac{1}{f(x)} \nabla f(x) \cdot x \)

\( \text{El}_a f(x) \) is approximately the percentage change in \( f(x) \) corresponding to a one percent increase in each component of \( x \). (See (4.27)–(4.28) for \( f'_a(x) \).)

5.17 \( \text{El}_a f(x) = \sum_{i=1}^{n} \text{El}_i f(x), \quad a = \frac{x}{\|x\|} \)

A useful fact (the passus equation).

The marginal rate of substitution (MRS) of \( y \) for \( x \) is

5.18 \( R_{yx} = \frac{f'_1(x, y)}{f'_2(x, y)}, \quad f(x, y) = c \)

\( R_{yx} \) is approximately how much one must add of \( y \) per unit of \( x \) removed to stay on the same level curve for \( f \).