Chapter 5

Elasticities. Elasticities of substitution

5.1 $$\mathrm{El}_x f(x) = \frac{x}{f(x)} f'(x) = \frac{x}{y}\frac{dy}{dx} = \frac{d(\ln y)}{d(\ln x)}$$

$\mathrm{El}_x f(x)$, the *elasticity* of $y = f(x)$ w.r.t. x, is approximately the percentage change in $f(x)$ corresponding to a one per cent increase in x.

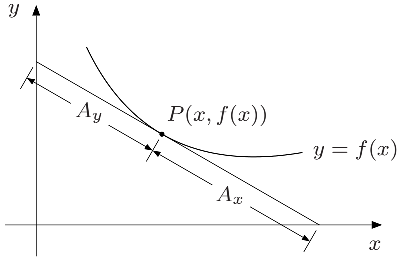
5.2 

Illustration of Marshall's rule.

5.3 *Marshall's rule:* To find the elasticity of $y = f(x)$ w.r.t. x at the point P in the figure, first draw the tangent to the curve at P. Measure the distance A_y from P to the point where the tangent intersects the y-axis, and the distance A_x from P to where the tangent intersects the x-axis. Then $\mathrm{El}_x f(x) = \pm A_y/A_x$.

Marshall's rule. The distances are measured positive. Choose the plus sign if the curve is increasing at P, the minus sign in the opposite case.

5.4
- If $|\mathrm{El}_x f(x)| > 1$, then f is elastic at x.
- If $|\mathrm{El}_x f(x)| = 1$, then f is unitary elastic at x.
- If $|\mathrm{El}_x f(x)| < 1$, then f is inelastic at x.
- If $|\mathrm{El}_x f(x)| = 0$, then f is completely inelastic at x.

Terminology used by many economists.

5.5 $$\mathrm{El}_x(f(x)g(x)) = \mathrm{El}_x f(x) + \mathrm{El}_x g(x)$$

General rules for calculating elasticities.

5.6 $$\mathrm{El}_x\left(\frac{f(x)}{g(x)}\right) = \mathrm{El}_x f(x) - \mathrm{El}_x g(x)$$

5.7	$\mathrm{El}_x(f(x) \pm g(x)) = \dfrac{f(x)\,\mathrm{El}_x f(x) \pm g(x)\,\mathrm{El}_x g(x)}{f(x) \pm g(x)}$	General rules for calculating elasticities.
5.8	$\mathrm{El}_x f(g(x)) = \mathrm{El}_u f(u)\,\mathrm{El}_x u, \quad u = g(x)$	
5.9	If $y = f(x)$ has an inverse function $x = g(y) = f^{-1}(y)$, then, with $y_0 = f(x_0)$, $\mathrm{El}_y x = \dfrac{y}{x}\dfrac{dx}{dy}, \quad \text{i.e.} \quad \mathrm{El}_y(g(y_0)) = \dfrac{1}{\mathrm{El}_x f(x_0)}$	The elasticity of the inverse function.
5.10	$\mathrm{El}_x A = 0, \quad \mathrm{El}_x x^a = a, \quad \mathrm{El}_x e^x = x.$ (A and a are constants, $A \neq 0$.)	Special rules for elasticities.
5.11	$\mathrm{El}_x \sin x = x \cot x, \quad \mathrm{El}_x \cos x = -x \tan x$	
5.12	$\mathrm{El}_x \tan x = \dfrac{x}{\sin x \cos x}, \quad \mathrm{El}_x \cot x = \dfrac{-x}{\sin x \cos x}$	
5.13	$\mathrm{El}_x \ln x = \dfrac{1}{\ln x}, \quad \mathrm{El}_x \log_a x = \dfrac{1}{\ln x}$	
5.14	$\mathrm{El}_i f(\mathbf{x}) = \mathrm{El}_{x_i} f(\mathbf{x}) = \dfrac{x_i}{f(\mathbf{x})}\dfrac{\partial f(\mathbf{x})}{\partial x_i}$	The *partial elasticity* of $f(\mathbf{x}) = f(x_1, \dots, x_n)$ w.r.t. x_i, $i = 1, \dots, n$.
5.15	If $z = F(x_1, \dots, x_n)$ and $x_i = f_i(t_1, \dots, t_m)$ for $i = 1, \dots, n$, then for all $j = 1, \dots, m$, $\mathrm{El}_{t_j} z = \sum_{i=1}^{n} \mathrm{El}_i F(x_1, \dots, x_n)\,\mathrm{El}_{t_j} x_i$	The *chain rule for elasticities.*
5.16	The *directional elasticity* of f at $\mathbf{x}$, in the direction of $\mathbf{x}/\lVert\mathbf{x}\rVert$, is $\mathrm{El}_{\mathbf{a}} f(\mathbf{x}) = \dfrac{\lVert\mathbf{x}\rVert}{f(\mathbf{x})} f'_{\mathbf{a}}(\mathbf{x}) = \dfrac{1}{f(\mathbf{x})} \nabla f(\mathbf{x}) \cdot \mathbf{x}$	$\mathrm{El}_{\mathbf{a}} f(\mathbf{x})$ is approximately the percentage change in $f(\mathbf{x})$ corresponding to a one per cent increase in each component of $\mathbf{x}$. (See (4.27)–(4.28) for $f'_{\mathbf{a}}(\mathbf{x})$.)
5.17	$\mathrm{El}_{\mathbf{a}} f(\mathbf{x}) = \sum_{i=1}^{n} \mathrm{El}_i f(\mathbf{x}), \quad \mathbf{a} = \dfrac{\mathbf{x}}{\lVert\mathbf{x}\rVert}$	A useful fact (the *passus equation*).
5.18	The *marginal rate of substitution* (MRS) of y for x is $R_{yx} = \dfrac{f'_1(x,y)}{f'_2(x,y)}, \qquad f(x,y) = c$	R_{yx} is approximately how much one must add of y per unit of x removed to stay on the same level curve for f.