Chapter 9

Integration

Indefinite integrals

9.1 $$\int f(x)\,dx = F(x) + C \iff F'(x) = f(x)$$ — Definition of the *indefinite integral.*

9.2 $$\int (af(x) + bg(x))\,dx = a\int f(x)\,dx + b\int g(x)\,dx$$ — *Linearity* of the integral. a and b are constants.

9.3 $$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$ — *Integration by parts.*

9.4 $$\int f(x)\,dx = \int f(g(t))g'(t)\,dt, \quad x = g(t)$$ — *Change of variable. (Integration by substitution.)*

9.5 $$\int x^n\,dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + C, & n \neq -1 \\ \ln|x| + C, & n = -1 \end{cases}$$ — Special integration results.

9.6 $$\int a^x\,dx = \frac{1}{\ln a}a^x + C, \quad a > 0,\ a \neq 1$$

9.7 $$\int e^x\,dx = e^x + C$$

9.8 $$\int xe^x\,dx = xe^x - e^x + C$$

9.9 $$\int x^n e^{ax}\,dx = \frac{x^n}{a}e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx,\ a \neq 0$$

9.10 $$\int \log_a x\,dx = x\log_a x - x\log_a e + C,\ a > 0,\ a \neq 1$$

9.11	$\int \ln x \, dx = x \ln x - x + C$	Special integration results.
9.12	$\int x^n \ln x \, dx = \frac{x^{n+1}\big((n+1)\ln x - 1\big)}{(n+1)^2} + C$	$(n \neq -1)$
9.13	$\int \sin x \, dx = -\cos x + C$	
9.14	$\int \cos x \, dx = \sin x + C$	
9.15	$\int \tan x \, dx = -\ln \lvert\cos x\rvert + C$	
9.16	$\int \cot x \, dx = \ln \lvert\sin x\rvert + C$	
9.17	$\int \frac{1}{\sin x} \, dx = \ln \left\lvert \frac{1-\cos x}{\sin x} \right\rvert + C$	
9.18	$\int \frac{1}{\cos x} \, dx = \ln \left\lvert \frac{1+\sin x}{\cos x} \right\rvert + C$	
9.19	$\int \frac{1}{\sin^2 x} \, dx = -\cot x + C$	
9.20	$\int \frac{1}{\cos^2 x} \, dx = \tan x + C$	
9.21	$\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{2}\sin x \cos x + C$	
9.22	$\int \cos^2 x \, dx = \frac{1}{2}x + \frac{1}{2}\sin x \cos x + C$	
9.23	$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$	$(n \neq 0)$
9.24	$\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$	$(n \neq 0)$