A New Algorithm Based on Fuzzy Gibbs Random Fields for Image Segmentation

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Abstract. In this paper a new unsupervised segmentation algorithm based on Fuzzy Gibbs Random Field (FGRF) is proposed. This algorithm, named as FGS, can deal with fuzziness and randomness simultaneously. A Classical Gibbs Random Field (CGRF) servers as bridge between prior FGRF and original image. The FGRF is equivalent to CGRF when no fuzziness is considered; therefore, the FGRF is obviously a generalization of the CGRF. The prior FGRF is described in the Potts model, whose parameter is estimated by the maximum pesudolikelihood (MPL) method. The segmentation results are obtained by fuzzifying the image, updating the membership of FGRF based on maximum a posteriori (MAP) criteria, and defuzzifying the image according to maximum membership principle (MMP). Specially, this algorithm can filter the noise effectively. The experiments show that this algorithm is obviously better than CGRF-based methods and conventional FCM methods as well.

1 Introduction

Image segmentation is a key technique in the pattern recognition, computer vision and image analysis. The accurately segmented medical images is very helpful for clinical diagnose and quantitative analysis. Automated segmentation is however very complicated, facing difficulties due to overlapping intensities, anatomical variability in shape, size, and orientation, partial volume effects, as well as noise perturbation, intensity inhomogeneities, and low contrast in images [1]. To overcome those difficulties, there has recently been growing interesting in soft segmentation methods [2]-[4]. The soft segmentation, where each pixel may be classified into classes with a varying degree of membership, is a more natural way. Introducing the fuzzy set theory into the segmentation is the outstanding contribution for soft segmentation algorithms. The algorithm is called soft image segmentation scheme if it is based on fuzzy set. When the fuzziness is eliminated according to some rules, the segmented image is exactly obtained.

Although the FCM algorithms are widely used in image segmentation, there are still some disadvantages. It is assumed that the data are of spatial independence or no

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context. Those assumptions are unreasonable. The FCM algorithm hardly deals with noised images. It is indispensable to incorporate the contextual constraint into the algorithm during segmentation. Another, the statistical approaches are increasingly used. Among them, Markov random fields (MRF)-based methods are of most importance due to well modeling the prior information [5], [6], but they poorly deal with fuzziness. Furthermore, only hard segmentation was obtained with these methods.

The fuzzy-based methods and MRF-based methods have their respective advantages. It can be predicated that integrating the fuzzy set theory with MRF theory will create wonderful results. The mixing of Markov and fuzzy approaches is discussed in [7]-[9]. Only two-class segmentation is discussed by adding a fuzzy class [7], [8]. H. Caillol and F. Salzenstein only have discussed about generalization to multi-class segmentation. S. Ruan et al used the fuzzy Markov Random Field (FMRF) as a prior to segmented MR medical image [9], which is a multi-class problem. However, only two-tissue mixtures are considered. Three-tissue or more-tissue mixtures are not concerned. The idea of merging the fuzziness and randomness is to be refreshed. The new concept of FMRF based on fuzzy random variable should be proposed. Every pixel is considered as fuzzy case, and is the mixture of all the classes.

The paper is organized as follows. In section 2 some preliminaries about our model are mentioned. The concept of FGRF is represented in Section 3. Our model based on FGRF is described in section 4. Section 5 gives the algorithm and some experiments. The final section is concerning conclusions and discussion on this paper.

2 Preliminaries

Fuzzy set theory is the extension of conventional set theory. It was introduced by Prof. Lotfi A. Zadeh of UC/Berkeley in 1965 to model the vagueness and ambiguity. Given a set $U$, a fuzzy set $\tilde{A}$ in $U$ is a mapping from $U$ into interval $[0,1]$, i.e.,

$$\tilde{A}: U \rightarrow [0,1], x \mapsto \tilde{A}(x),$$

where $\tilde{A}(x)$ is called membership function. Given $x_0 \in X$, $\tilde{A}(x_0)$ is the membership value for element $x_0$. All fuzzy sets in $U$ are denoted by $\mathbb{F}(U)$.

Fuzzy set $\tilde{A}$ in $U$ is denoted by $\tilde{A} = \{(x, \tilde{A}(x)) | x \in U\}$. If the set $U$ is finite, then fuzzy set $\tilde{A}$ can be written as $\tilde{A} = \sum \tilde{A}(x_i)/x_i$ or as a fuzzy vector $\tilde{A} = (\tilde{A}(x_1), \tilde{A}(x_2), \ldots, \tilde{A}(x_n))$. We always consider that the fuzzy set is the same notation as its membership function.

Uncertainty of data comes from fuzziness and randomness modeled by the random variables and the fuzzy sets respectively. Fuzzy random variable can model the two kinds of uncertainty first proposed by Kwakernaak [10]. In the definition, fuzzy random variable is a fuzzy-valued mapping. It is necessary to define the probability of fuzzy event, thereby.

The probability of fuzzy events $\tilde{A}: \Omega \rightarrow [0,1], \omega \mapsto \tilde{A}(\omega)$ in the probability space $(\Omega, \mathbb{F}, P)$ is defined as

$$P(\tilde{A}) = E(\tilde{A}(\omega)) = \int_{\Omega} \tilde{A}(\omega) P(d\omega),$$