An Application of Game-Refinement Theory to Mah Jong

Hiroyuki Iida¹,³, Kazutoshi Takahara¹, Jun Nagashima¹, Yoichiro Kajihara¹, and Tsuyoshi Hashimoto²

¹Department of Computer Science, Shizuoka University, 3-5-1 Johoku Hamamatsu, 432-8011 Japan
{iida, cs9055, cs8066, cs6501}@cs.inf.shizuoka.ac.jp
²Department of Systems Engineering, Shizuoka University, 3-5-1 Johoku Hamamatsu, 432-8011 Japan
hasimoto@cs.inf.shizuoka.ac.jp
³Information and Systems, PRESTO, Japan Science and Technology Agency, Japan

Abstract. This paper presents an application of the game refinement theory to a class of multi-person incomplete-information games, especially in the domain of Mah Jong that is an old Chinese four-person incomplete-information game. We have developed a computer program to analyze some statistics on the number of the possible options and the game length. The results of the analysis show that the measure of the game-refinement for Mah Jong has an appropriate value as well as other refined games such as chess and Go.

Keywords: game-refinement theory, multi-person games with incomplete-information, and Mah Jong

1 Introduction

We have studied the property of the decision space in game playing [2] [3] [5] [6]. The decision space is the minimal search space without forecasting. It provides the common measures for almost all board games. The dynamics of decision options in the decision space has been investigated and we observed that this dynamics was a key factor for game entertainment. Then we proposed the measure of the refinement in games [3].

Interesting games are always uncertain until the last end of games. Thus the variation in available options stays constant all over games. Here the games are a kind of seesaw game between possible results. In contrast, one player quickly dominates over the other in uninteresting games. Here options are likely to be diminishing quickly. Therefore, the refined games are more likely to be in a seesaw game. We then call the principle of seesaw games.

Based on the principle of seesaw games, we proposed a logistic model of game uncertainty [4]. From the players' viewpoint, the information on the game result is an increasing function of time (the number of moves) \( t \). We here define the information on the game result as the amount of solved uncertainty \( x(t) \), such that
\[ x'(t) = \frac{n}{t} x(t) \]  \hspace{1cm} (1)

where constant \( n \) is the expected number of plausible moves that is determined based on the ability difference between the two players of a game, and \( x(0) = 0 \) and \( x(D) = B \). Note that \( 0 \leq t \leq D, 0 \leq x(t) \leq B \). The above equation implies that the rate of increase in the solved information \( x'(t) \) is proportional to \( x(t) \) and inverse proportional to \( t \). Solving Equation (1), we get

\[ x(t) = B \left( \frac{t}{D} \right)^n \]  \hspace{1cm} (2)

Here we assume that the solved information \( x(t) \) is twice derivable at \( t \in [0, D] \). The second derivative here indicates the accelerated velocity of the solved uncertainty along the game progress. It is the difference of the rate of acquired information during game progress.

\[ x''(t) = \frac{B}{D^n} n(n-1)t^{n-2} \]  \hspace{1cm} (3)

A good dynamic seesaw game in which the result is unpredictable at the very last moves in the endgame stage corresponds with a high value of the second derivative at \( t = D \). This implies that game is more exciting, fascinating and entertaining if this value is larger. We expect that this property is the most important characteristics of a well-refined game.

At \( t = D \) (the last few moves of an endgame), Equation (3) becomes:

\[ x''(D) = \frac{B}{D^n} n(n-1)D^{n-2} = \frac{B}{D^2} n(n-1) \]  \hspace{1cm} (4)

In the second derivative of \( x''(t) \), \( n \) is the constant related to the ability of players, or its root square \( \sqrt{\frac{B}{D}} \) is the value related to the game property.

This measure should reflect some aspects of the attractiveness of games. We then compared a class of board games by means of this measure. We especially compare various chess variants by means of this measure and other characteristics [3]. The measure was \( \sqrt{\frac{B}{D}} \) where \( B \) stands for the average number of possible moves and \( D \) stands for the average game length.

The rules and details of a game should have changed over the long history of games in most games and the current games are the evolutionary outcomes of a long history from the original games. For example, many variants of chess-like board games are known in history and the modern chess is the descendant of these variants.