Vorticity and Vortices in Geophysical Flows

Geophysical fluid dynamics studies the motion of the atmosphere and oceans that cover the earth, which is the basis of meteorology and oceanography, where the vorticity and circulation are also among the most basic concepts as pioneered by the work of Bjerknes (1898, 1902). The self-rotation of the earth produces the Coriolis force, which brings in many unique and attractive topics to the vorticity dynamics and vortex motion in rotating fluid dynamics. In particular, the conservation of the potential vorticity is so fundamental that to a large extent the geophysical fluid dynamics may be regarded as the dynamics of the potential-vorticity conservation.

Moreover, the coherent vortical structures in a rotating fluid are relevant to large-scale geophysical flows, and hence to the understanding of atmospheric and oceanic circulations as well as turbulence therein. Thus, it is important to investigate isolated vortices, including their dynamics, instability properties, mutual interaction, and motion in rotating fluid. Owing to the Coriolis force and approximate two-dimensionality of geophysical flows, in several basic aspects these structures and interactions are quite different from those in a three-dimensional flow without system rotation as treated in preceding chapters.

This chapter is an introduction to geophysical vorticity and vortex dynamics. We start from the basic equations in rotating and density-stratified fluid, discussing relevant dimensional parameters, scalings, and a few frequently used simplified models derived thereby. The concept of potential vorticity is then introduced and some of its most significant applications are illustrated. The evolution process of vortical structures, as well as the structures of barotropic and baroclinic vortices, are discussed later within the two-dimensional approximation. We finally discuss the motion of strong large-scale atmospheric vortices. For more comprehensive and in-depth materials of some of these topics, the interested reader may consult, e.g. Gill (1982a), Bengtsson and Lighthill (1982), Hoskins et al. (1985), Pedlosky (1987), Hopfinger and van Heijst (1993), Voropayev and Afanasyev (1994), Salmon (1998), and McWilliams (2005).
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12.1 Governing Equations and Approximations

12.1.1 Effects of Frame Rotation and Density Stratification

The fluids in the atmosphere and oceans are weakly density-stratified, with the density $\rho$ varying as the height when the fluid is at rest. For the atmosphere we use the perfect-gas equation $p = \rho RT$ where $R$ is the gas constant. For seawater the equation of state can be approximated by $\rho = \rho_0 [1 - \alpha_1 (T - T_0) + \alpha_2 (S - S_0)]$, where $S$ is the salinity (grams/kilograms), and $\alpha_1$ and $\alpha_2$ are empirically determined constants. For large-scale fluid motion under gravity, we may assume incompressibility, but general thermodynamic relations introduced in Sect. 2.3.3 are applicable whenever necessary. Then, in an inertial frame of reference ("absolute frame") $\Sigma$, the continuity equation and momentum equation read, respectively,

\[
\frac{D\rho}{Dt} = 0 \quad \text{or} \quad \frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = 0, \quad (12.1)
\]

\[
\frac{Du}{Dt} = -\frac{1}{\rho} \nabla p + \mathbf{g} + \nu \nabla^2 \mathbf{u}, \quad (12.2)
\]

where $\mathbf{g}$ is the gravitational acceleration at the earth surface. In geophysics, however, the flows are observed in a frame of reference $\Sigma'$ fixed to the earth, which has angular velocity $\Omega = \Omega \mathbf{k}$, with $\mathbf{k}$ being the unit vector along the rotating axis. To establish the hydrodynamic equations in such a rotating frame, we first recall the relation between the relative acceleration viewed in $\Sigma'$ and the "absolute acceleration" in an inertial frame $\Sigma$. Let a vector $\mathbf{e}$ of constant magnitude rotate with angular velocity $\Omega$, such that $\frac{d\mathbf{e}}{dt} = \Omega \times \mathbf{e}$. (12.3)

Thus, if $\Sigma'$ rotates about a point $O$ with angular velocity $\Omega$ in which the orthonormal basis vectors are $\mathbf{e}'_i$ ($i = 1, 2, 3$), then the rate of change of an arbitrary vector $\mathbf{b}$, appearing as $\mathbf{b}'_i \mathbf{e}'_i$ in $\Sigma'$, reads

\[
\frac{dB}{dt} = \frac{dB'}{dt} \mathbf{e}'_i + b'_i \frac{d\mathbf{e}'_i}{dt} = \left(\frac{dB}{dt}\right)_r + \Omega \times \mathbf{b}, \quad (12.4)
\]

where $(dB/dt)_r$ is the relative rate of change of $\mathbf{b}$. Denoting $d\Omega/dt$ by $\dot{\Omega}$, and applying (12.4) to the position vector $\mathbf{r}$ of a fluid element from $O$ and the "absolute velocity" $\mathbf{u}$, we have

\[
\frac{dr}{dt} = \left(\frac{dr}{dt}\right)_r + \Omega \times \mathbf{r} \quad \text{or} \quad \mathbf{u} = \mathbf{u}' + \Omega \times \mathbf{r}, \quad (12.5)
\]

\[
\frac{Du}{Dt} = \left(\frac{Du}{Dt}\right)_r + \Omega \times \mathbf{u} + \dot{\Omega} \times \mathbf{r}, \quad (12.6)
\]